

Mapas de la Resonancia Secular ν_{\odot} para Satélites Irregulares Retrógrados

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■ Introducción

Satélites Irregulares al 2/2010

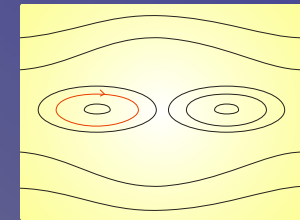
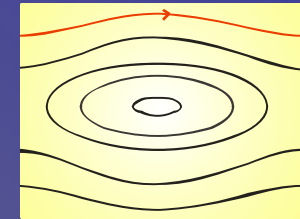
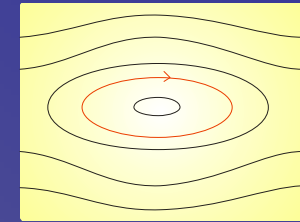
Júpiter 55

Saturno 38

Urano 9

Neptuno 6

$$\bar{\omega} - \bar{\omega}_\alpha$$



■ Whipple & Shelus (1993)

■ Saha & Tremaine (1993)

■ Nesvorný et al.(2003)

■ Cuk & Burns (2004)

■ Beaugé & Nesvorný (2007)

Pasiphae(J⁻)

Sinope(J⁻)

Pasiphae

Siarnaq(S⁺) – Sthepano(U⁻)

Narvi(S⁻)

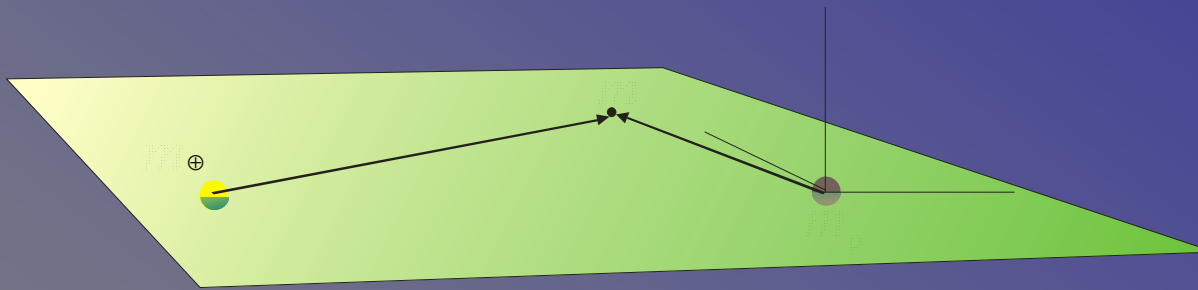
libra (0°)

libra (0°)-circ.

circulación

libración asimétrica

Modelos Analíticos para Resonancia...



Variables canónicas Delaunay

$$\begin{aligned} L &= \sqrt{\mu a} & ; & M \\ G &= L\sqrt{1-e^2} & ; & \omega \\ H &= G \cos i & ; & \Omega. \end{aligned}$$

$$F = F_0(L, \Lambda) + R(L, G, H, M, \omega, \Omega, M_{\alpha})$$

P2C

Perturbación

Yokoyama et al. (2003)

$$F_1(G^*, H^*, \omega^*, \Omega^*; \alpha^*)$$

$$\alpha^* = \frac{L^{*2}}{\mu a_{\alpha}}$$

variable de interés



$$\theta = \varpi - \varpi_{\alpha}$$

órbitas

directas

$$i < 90^{\circ}$$

$$\varpi = \Omega + \omega$$

órbitas

retrógradas

$$i > 90^{\circ}$$

$$\varpi = \Omega - \omega$$

variables “medias”

$$\begin{aligned} I_1 &= -G^* & ; & \quad \phi = \Omega^* - \omega^* \\ I_2 &= G^* + H^* & ; & \quad Q = \Omega^* \end{aligned}$$

$$\theta = \phi - \varpi_{\alpha}$$

variables “medias-medias”

T.C.

Nuevo Hamiltoniano

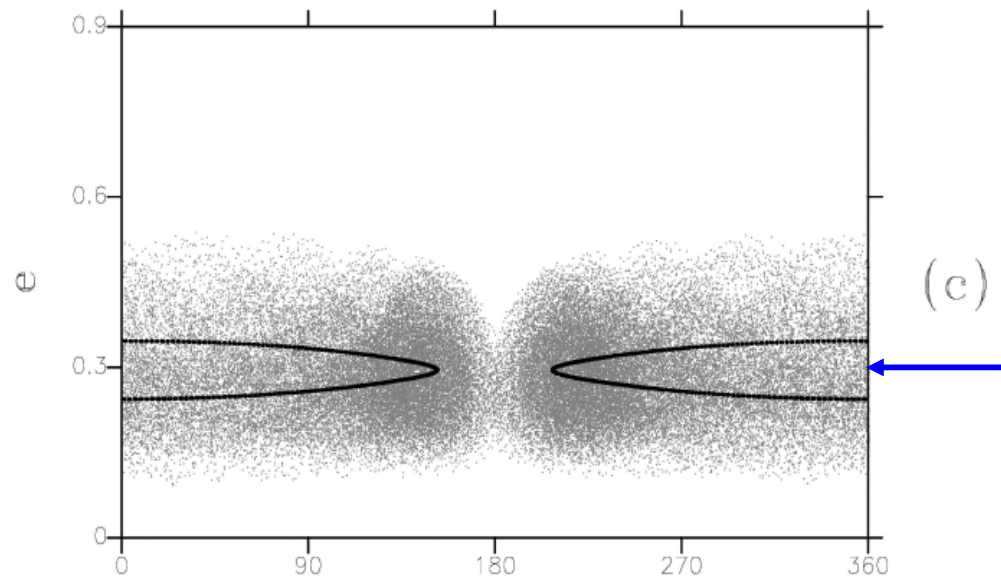
$$(I_1, I_2, \theta, Q) \rightarrow (I_1^*, I_2^*, \theta^*, Q^*)$$

$$F_1(I_1, I_2, \theta, Q) \rightarrow F_1^*(I_1^*, I_2^*, \theta^*)$$

$$F_1^* = A_0 + A_1 \cos(\theta^*) + A_2 \cos(2\theta^*)$$

$$A_k(I_1^*, I_2^*, L)$$

Dinámica secular de Sinope

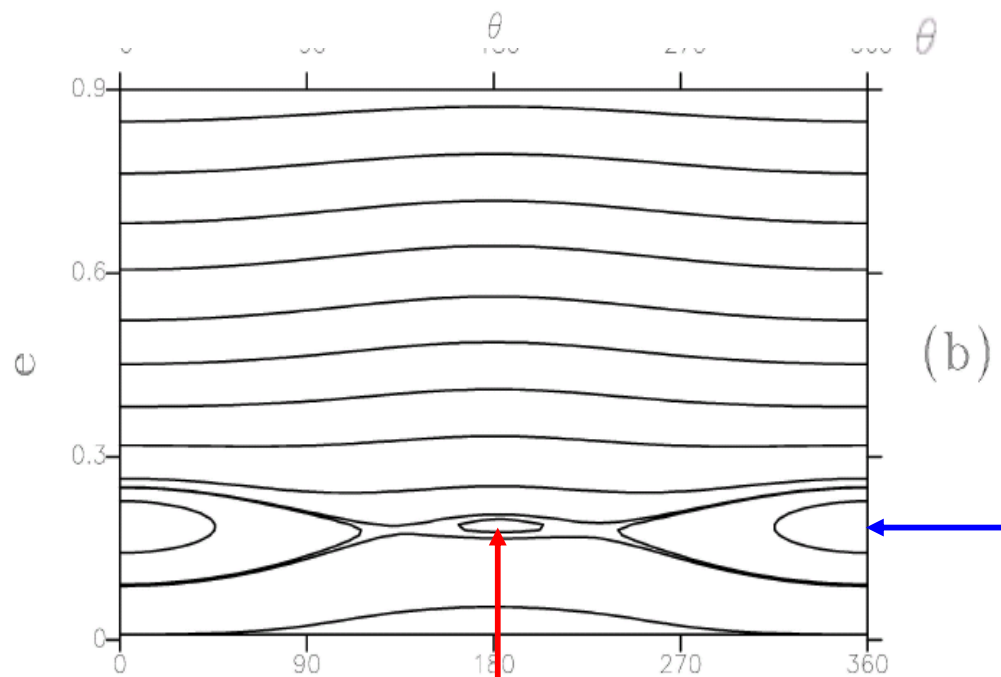


(c)

$$e^* = e^*(I_1^*, L^*, I_2^*)$$

$$L^* = 2.115 \times 10^{-4}$$

$$I_2^* = 1.575 \times 10^{-5}$$

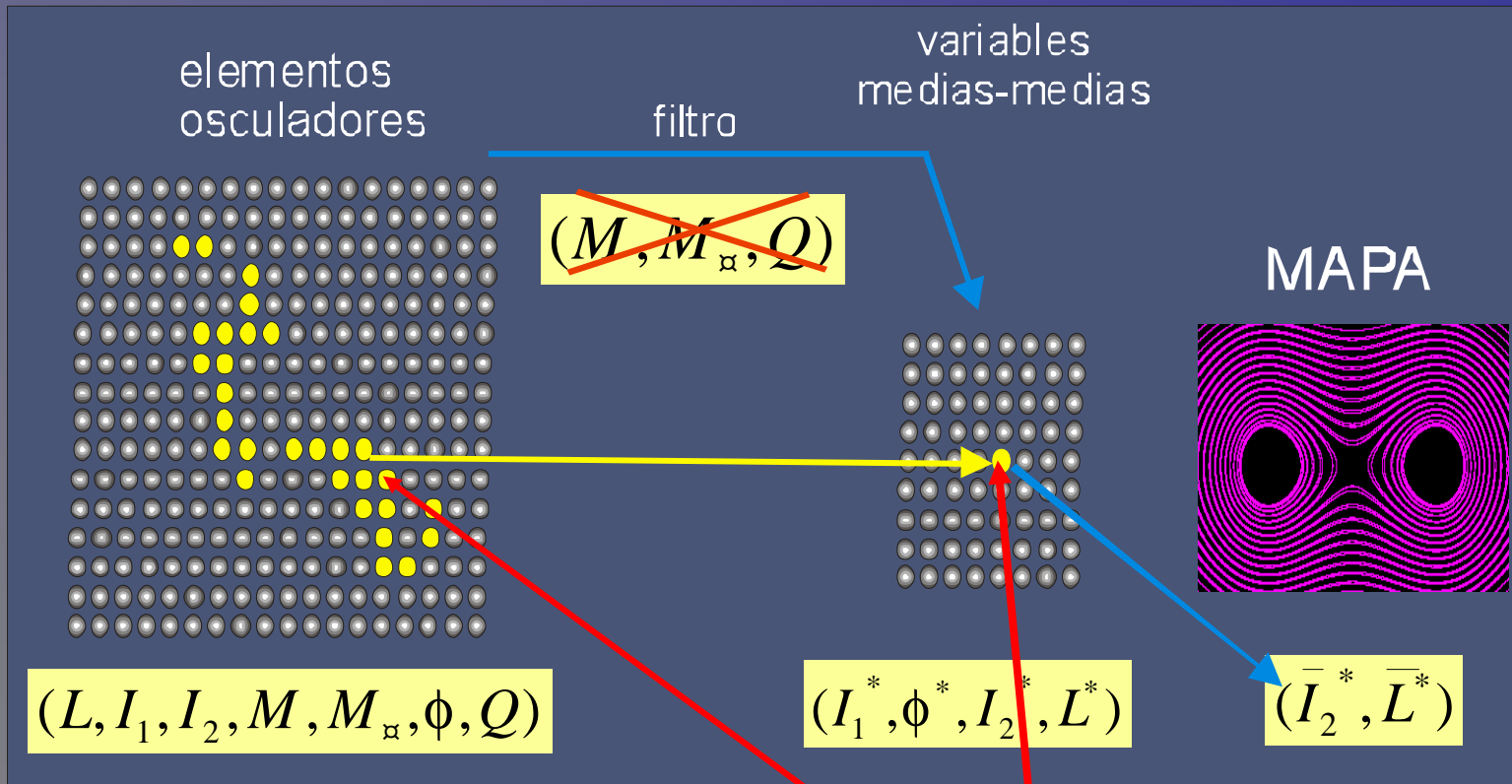


(b)

Beaugé & Nesvorný (2007)

$$F_3(G^*, H^*, \omega^*, \Omega^*; \alpha^*)$$

Implementando el Modelo Numérico



$$L^* = L + \frac{\partial}{\partial M} \chi = f_1(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

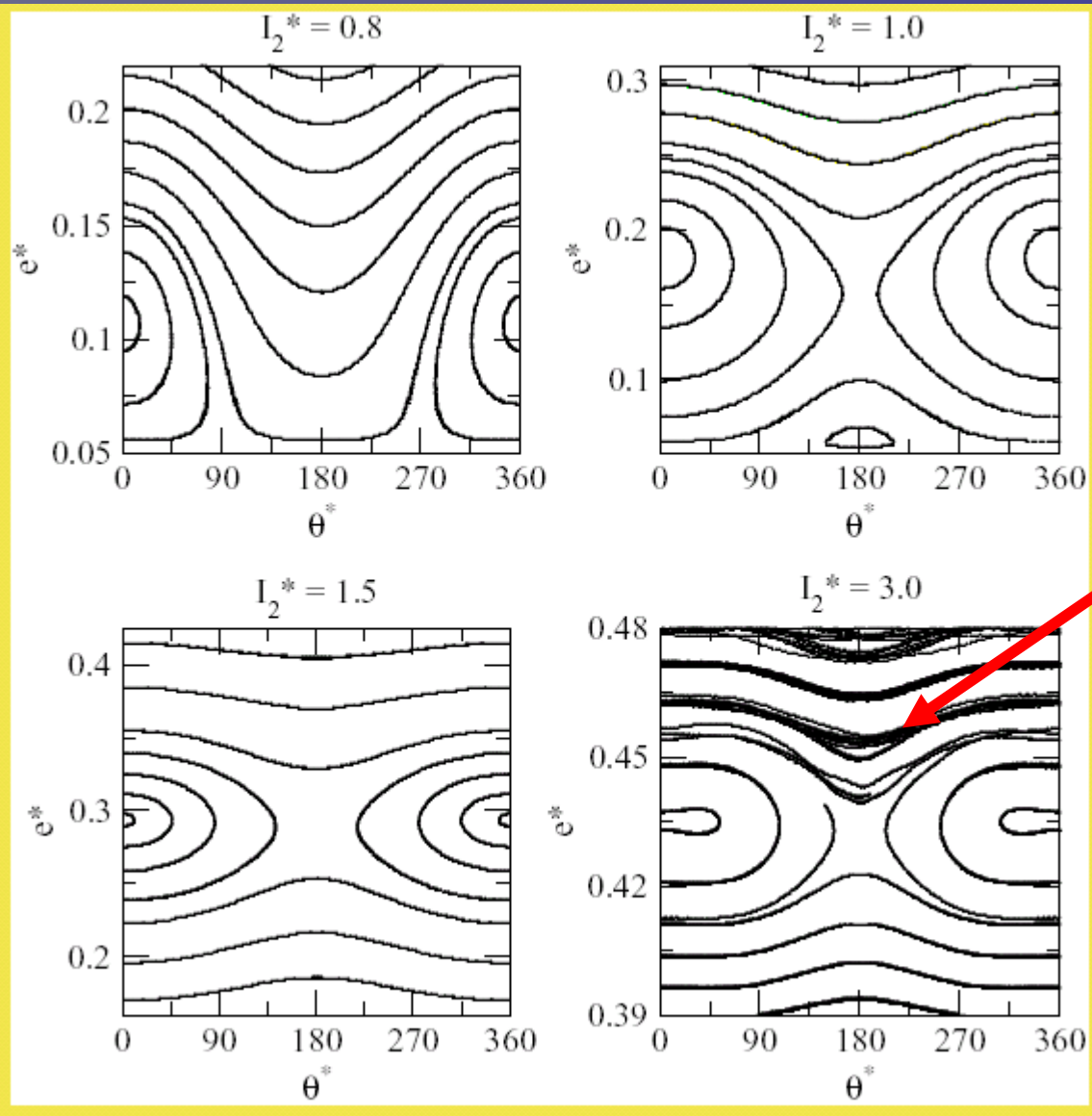
$$I_2^* = I_2 + \frac{\partial}{\partial Q} \chi = f_2(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$\chi = \chi(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$

$$L^* + \Delta L^* = L^* + \frac{\partial f_1}{\partial a} \Delta a + \frac{\partial f_1}{\partial i} \Delta i = \bar{L}^*$$

$$I_2^* + \Delta I_2^* = I_2^* + \frac{\partial f_2}{\partial a} \Delta a + \frac{\partial f_2}{\partial i} \Delta i = \bar{I}_2^*$$

Mapas de la Resonancia... (Sinope)



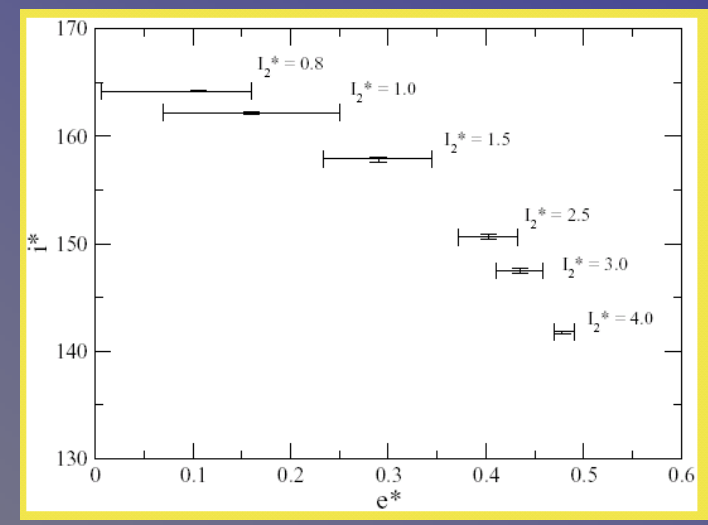
$$a^* = 0.16UA$$

$$L^* = 2.115 \times 10^{-4}$$

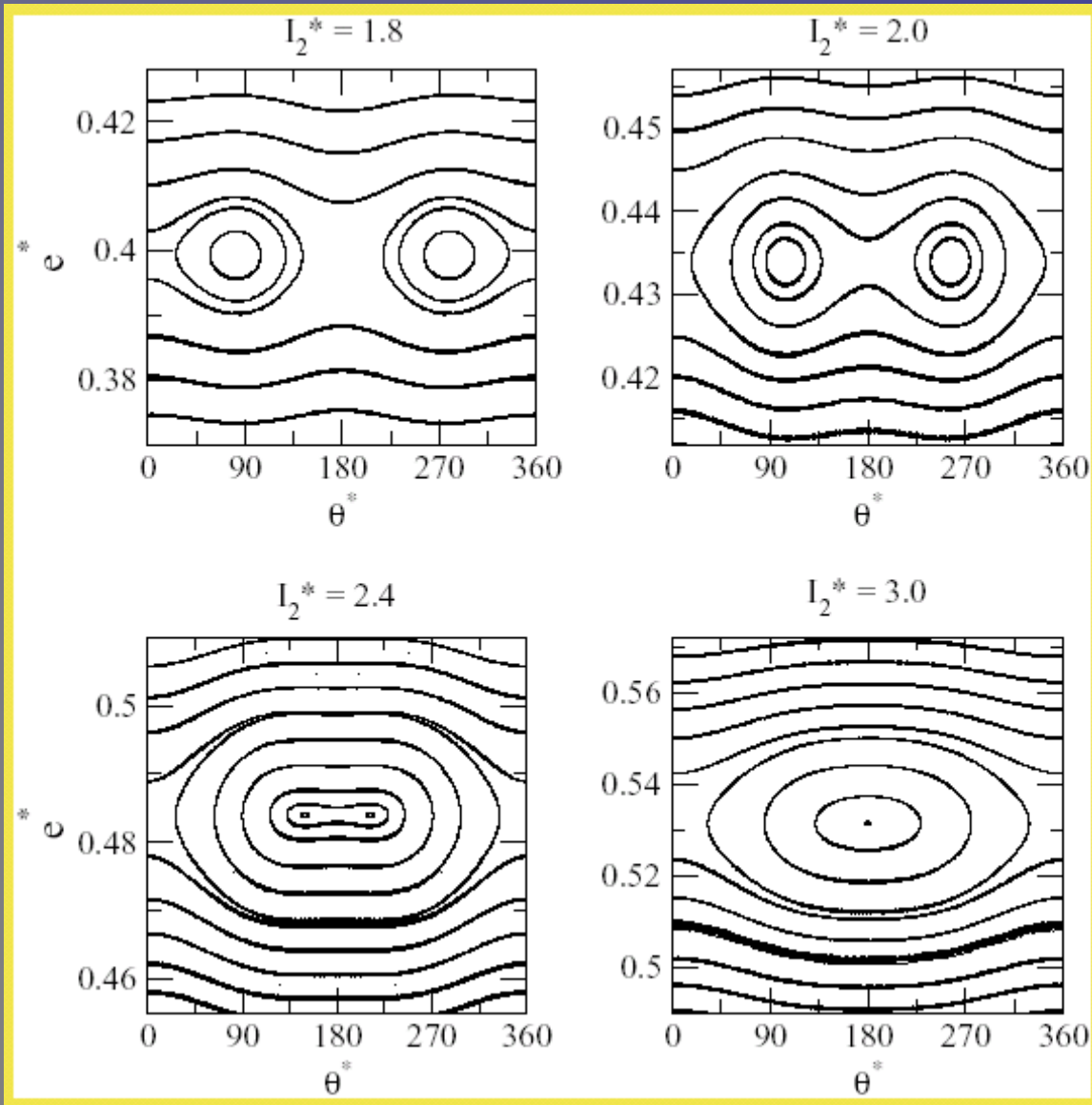
$$I_1^* = -G^* = -L^* \sqrt{1 - e^{*2}}$$

$$I_2^* = G^* + H^* = I_1^* (1 + \cos i^*)$$

RMM 6:1
Hinse et al. (2009)



Mapas de la Resonancia en Saturno...

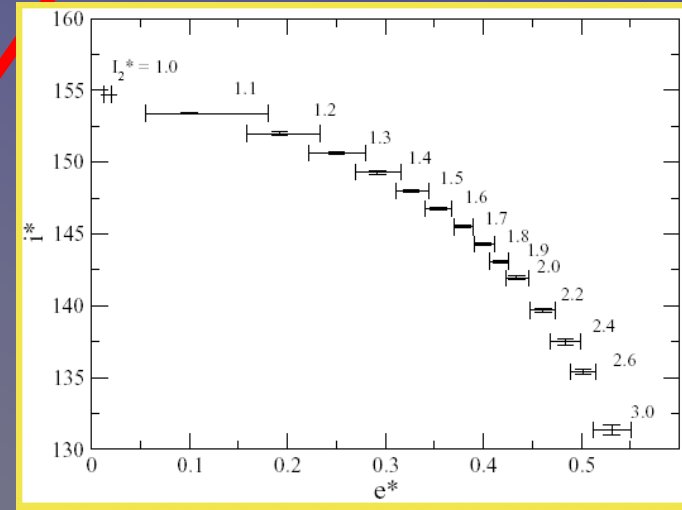


Narvi

$$a^* = 0.13UA$$

$$L^* = 1.040 \times 10^{-4}$$

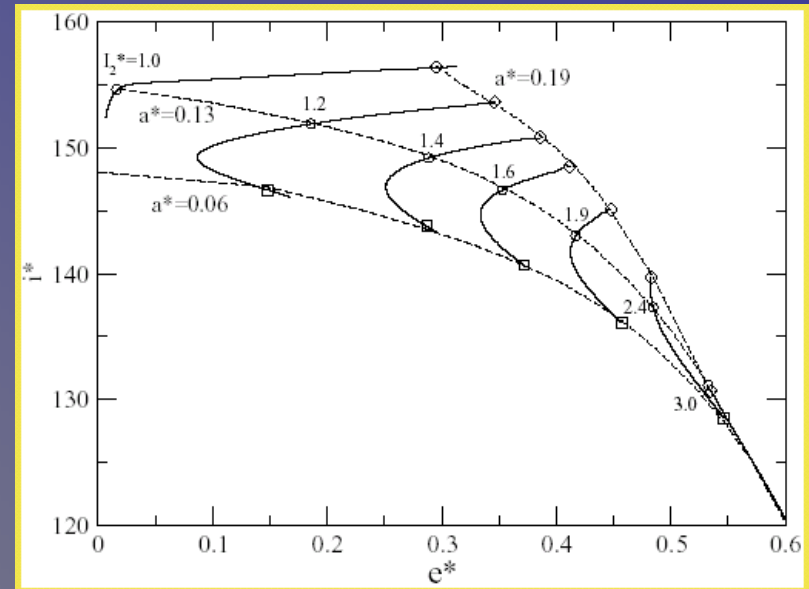
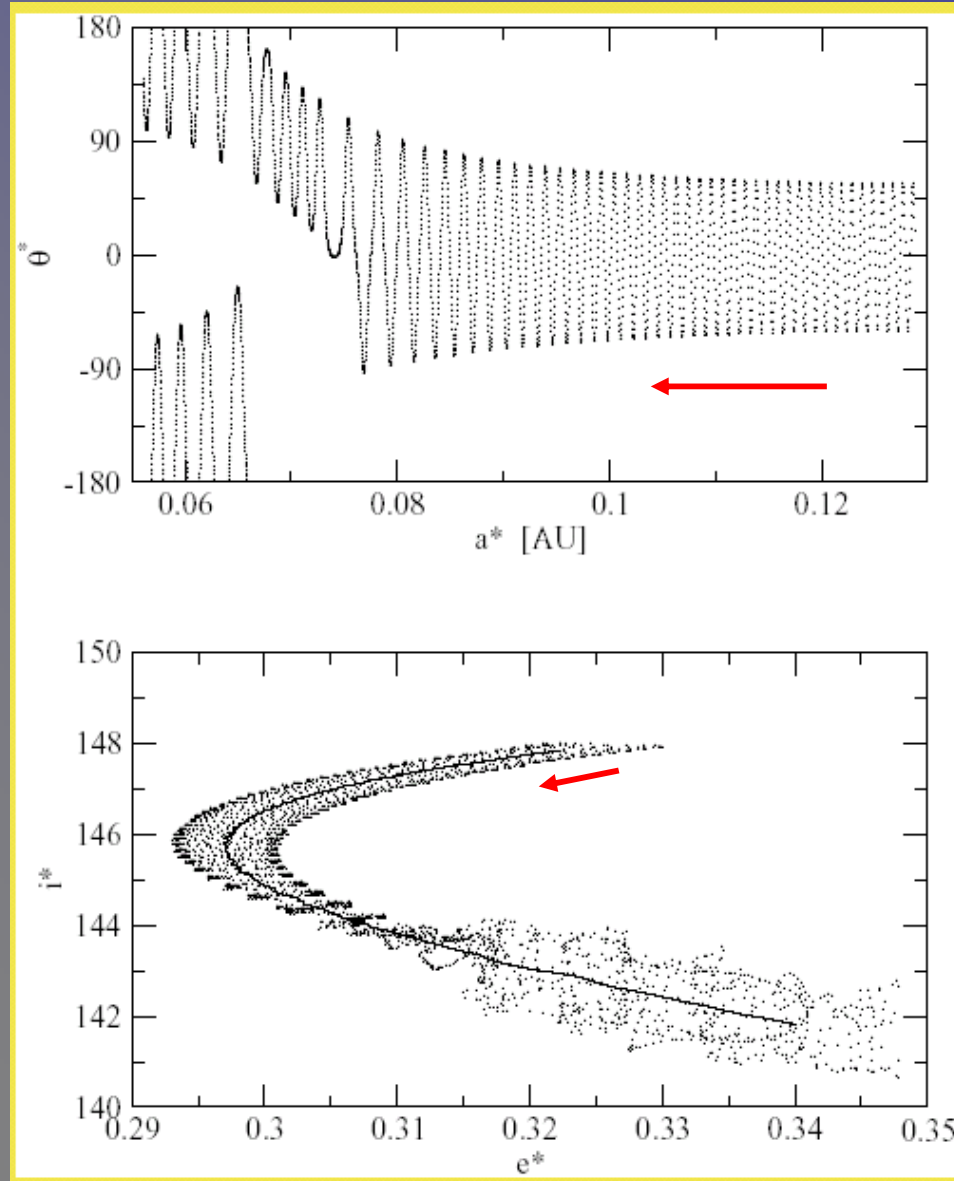
**libraciones
asimétricas**



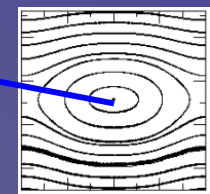
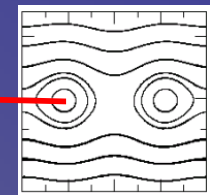
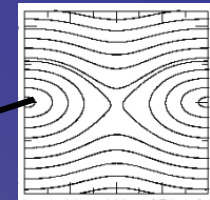
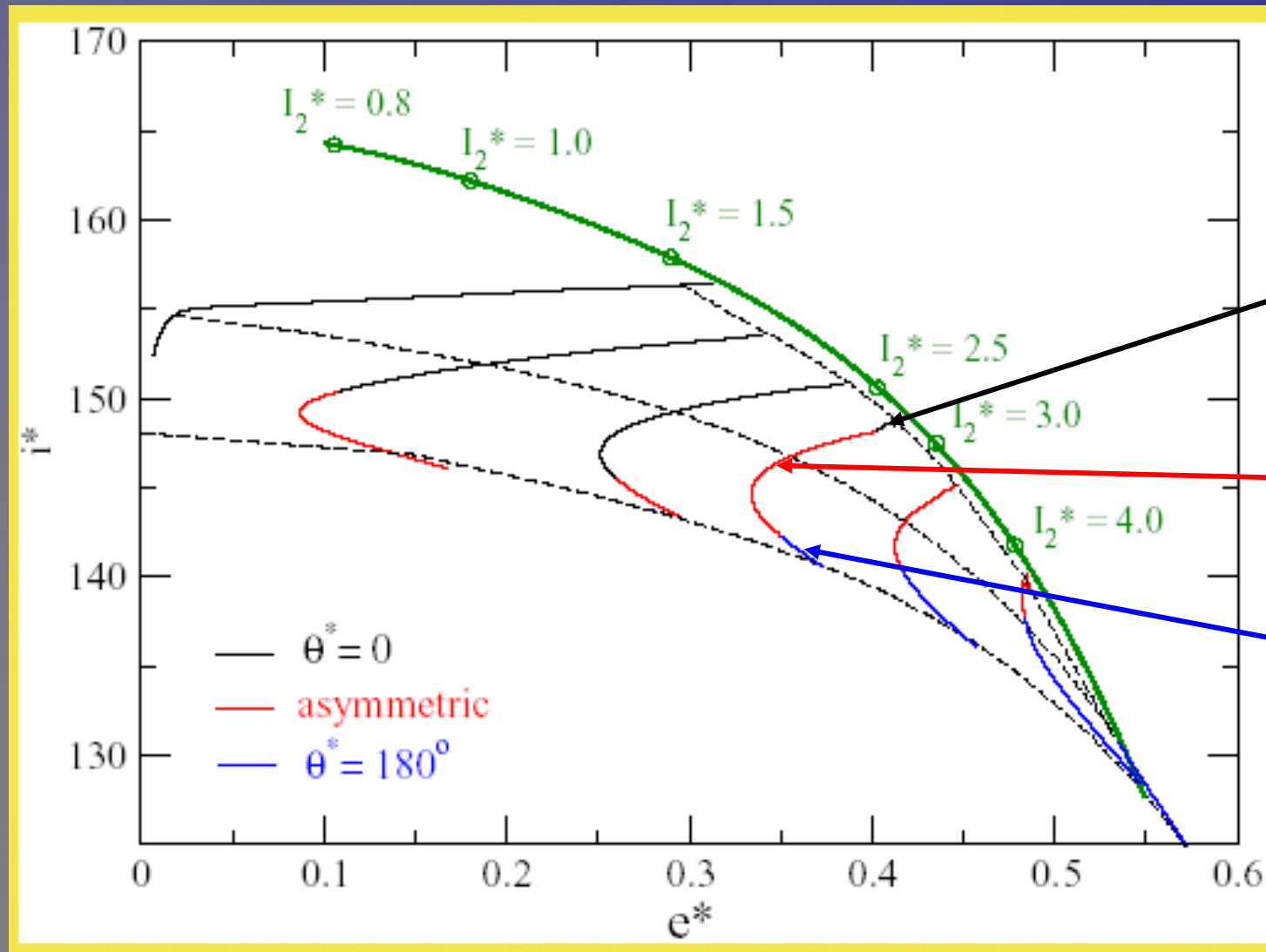
Utilizando migración adiabática

a^*

$\tau_a = 10^7$ años



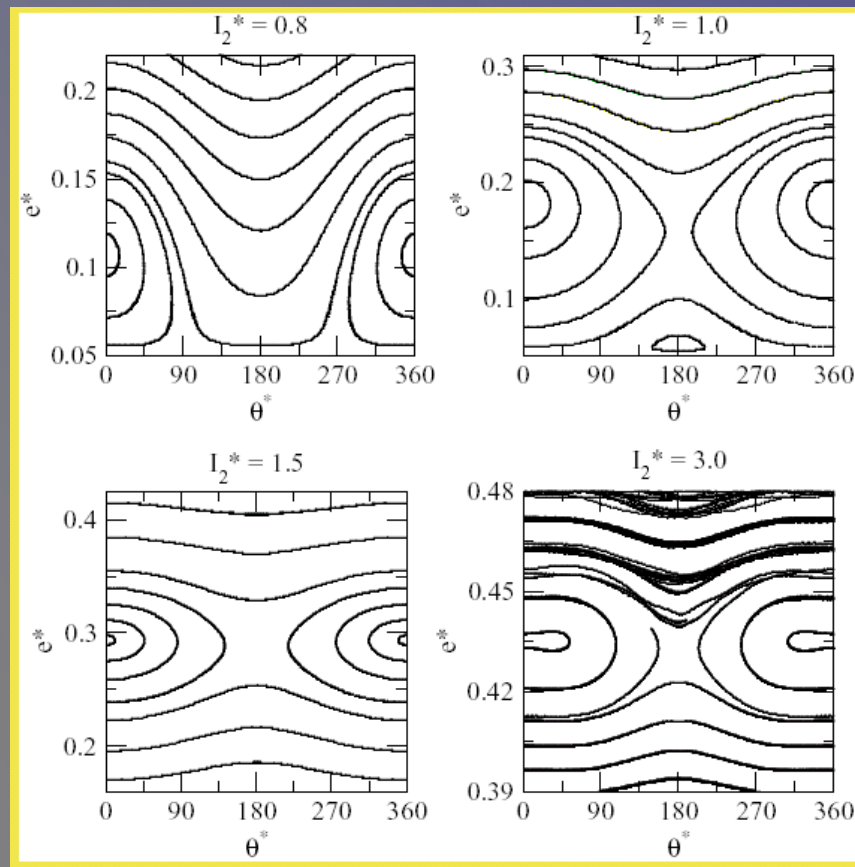
Familias de soluciones resonantes de amplitud-cero



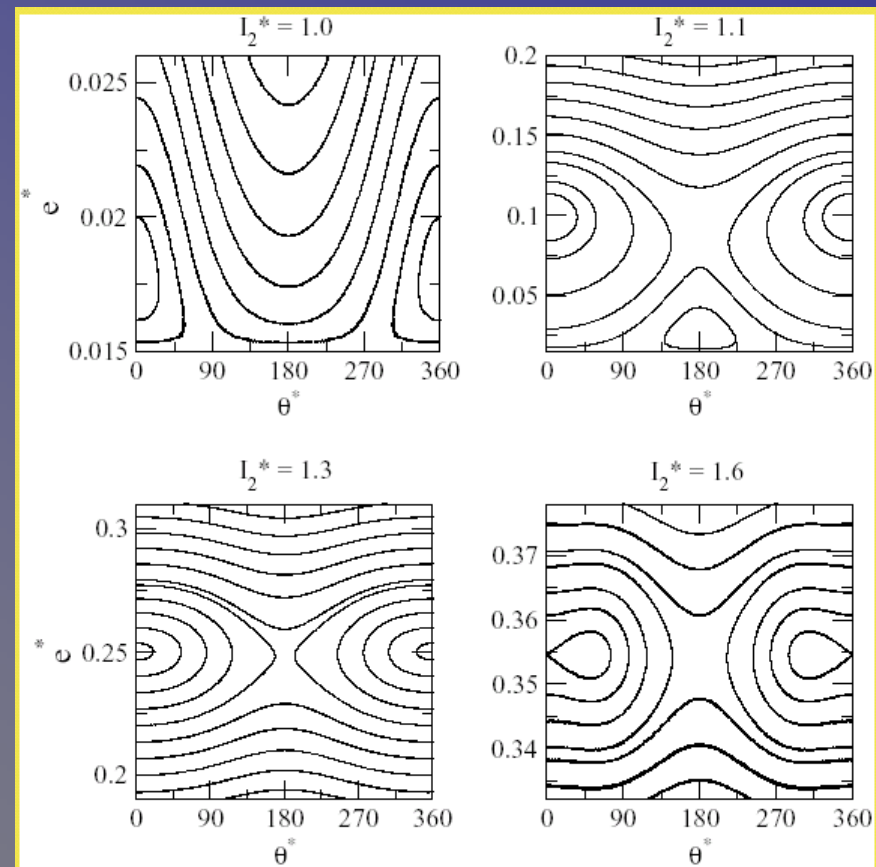
Factor de escala para otros sistemas de satélites

Teoría del Globo...

Júpiter



Saturno

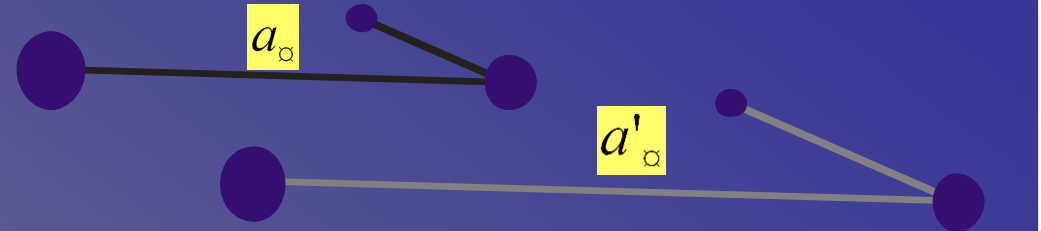


Principales efectos

■ Escala de distancias

$$\frac{a^*}{a_{\alpha}} \rightarrow \frac{a'^*}{a'_{\alpha}}$$

Las ecuaciones de movimiento quedan invariantes...



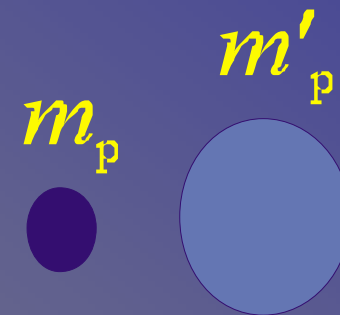
$$\Gamma_a = \frac{a'_{\alpha}}{a_{\alpha}}$$



$$a'^* = \Gamma_a a^*$$

■ Factor de masa Aproximación de Hill

$$\vec{r} \rightarrow \bar{\mu}^{\frac{1}{3}} \vec{r}$$



$$\Gamma_m = \left(\frac{m'_p}{m_p} \right)^{\frac{1}{3}}$$

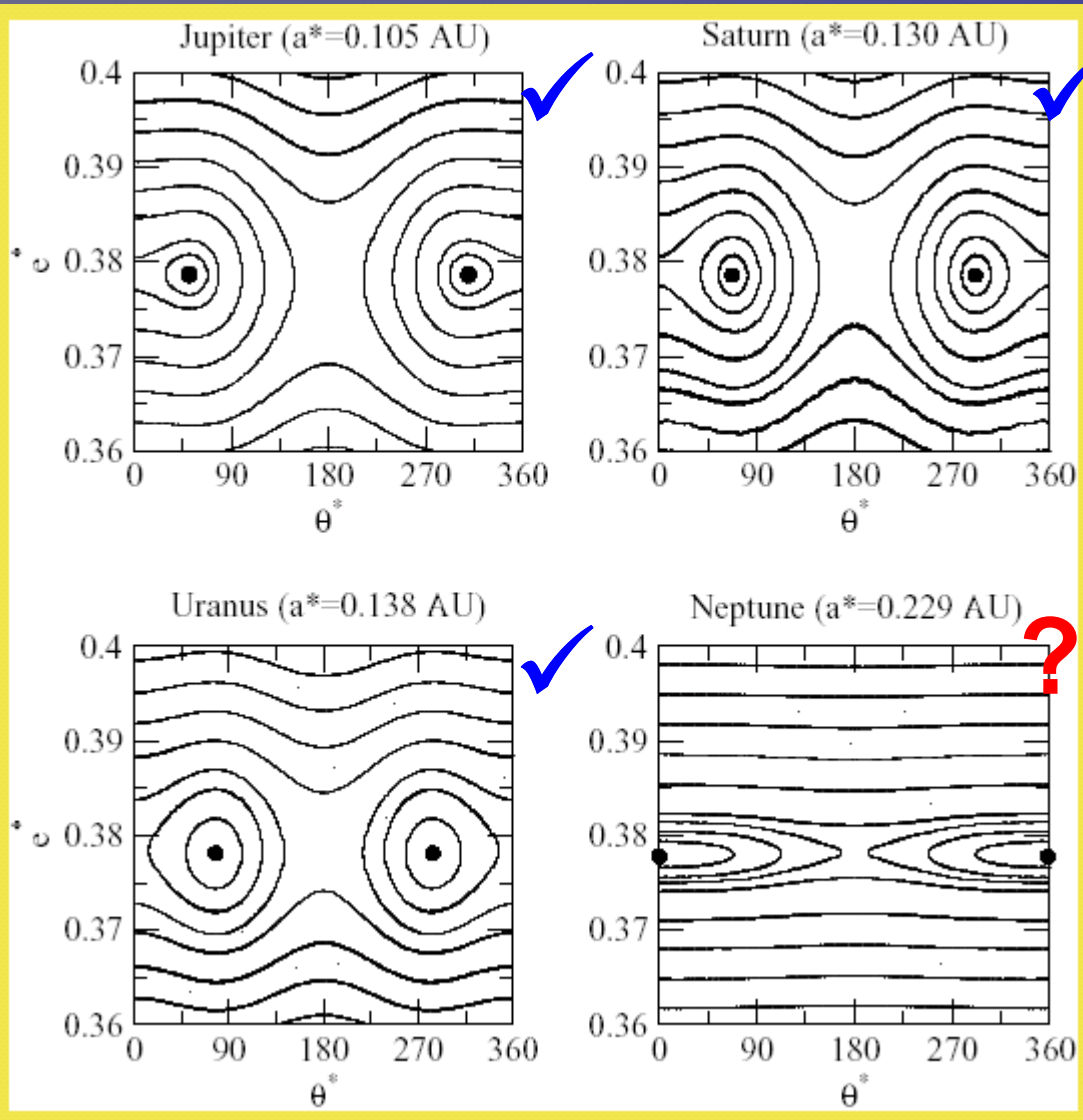


$$m'_p = \Gamma_m^3 m_p$$

FACTOR GLOBAL

$$a'^* = \Gamma_a \Gamma_m a^*$$

Resultados....



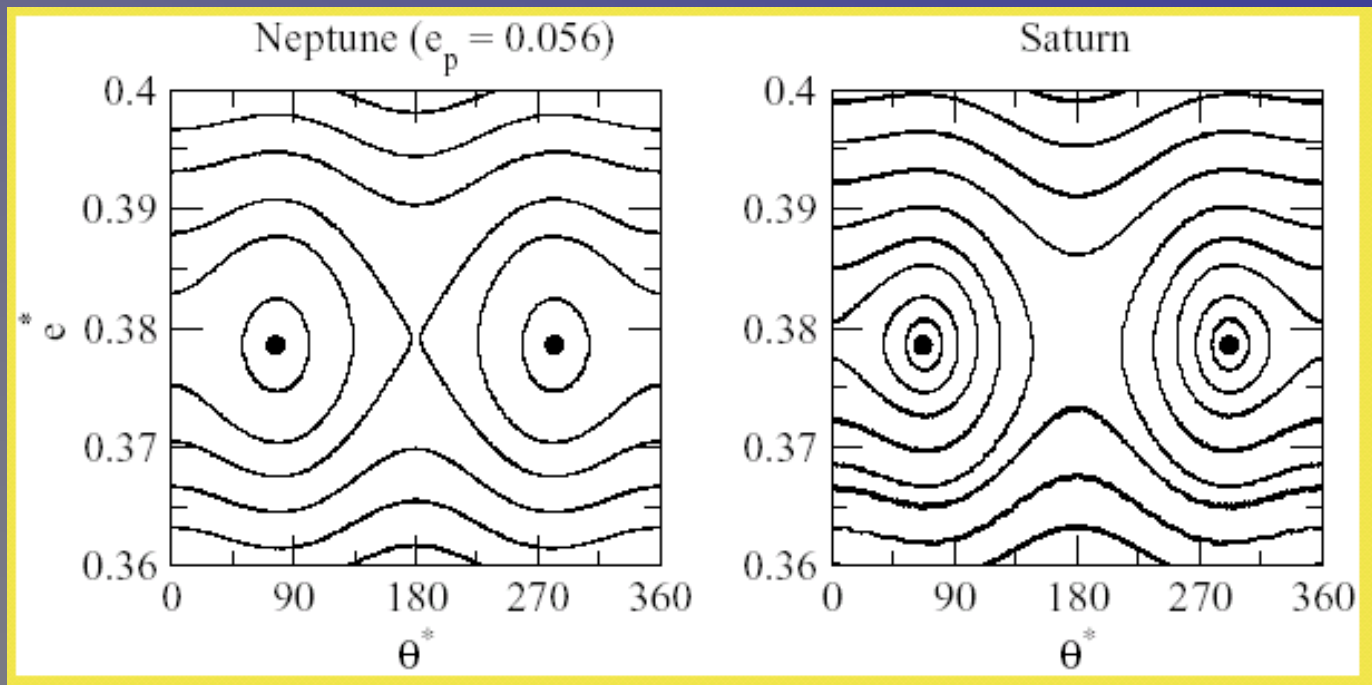
$$e_{Sat} > e_{Jup} > e_{Ura} > e_{Nep}$$

$$F_1^* = C_0 + C_1 e_{\alpha} \cos \theta^* + C_2 e_{\alpha}^2 \cos 2\theta^*$$



$$\sin \theta^* (C_1 + 4C_2 e_{\alpha} \cos \theta^*) = 0$$

$$4|C_2| e_{\alpha} > |C_1|$$



Conclusiones

- Dominio resonante con diversos modos de libración
- Modos restringidos a ciertos intervalos de i^* media-media

$$\theta = 0^0 \quad (i \geq 150^0)$$

$$\theta = 180^0 \quad (i \leq 140^0)$$

libraciones asimétricas en valores intermedios

- Buen acuerdo con la realidad (Pasiphae-Sinope-Narvi)
- Presentación de familias de soluciones resonantes (migración)
- Factor de escala de Mapas.

- Problema de Tres Cuerpos Restringido
- satélites retrógrados

Muchas gracias...

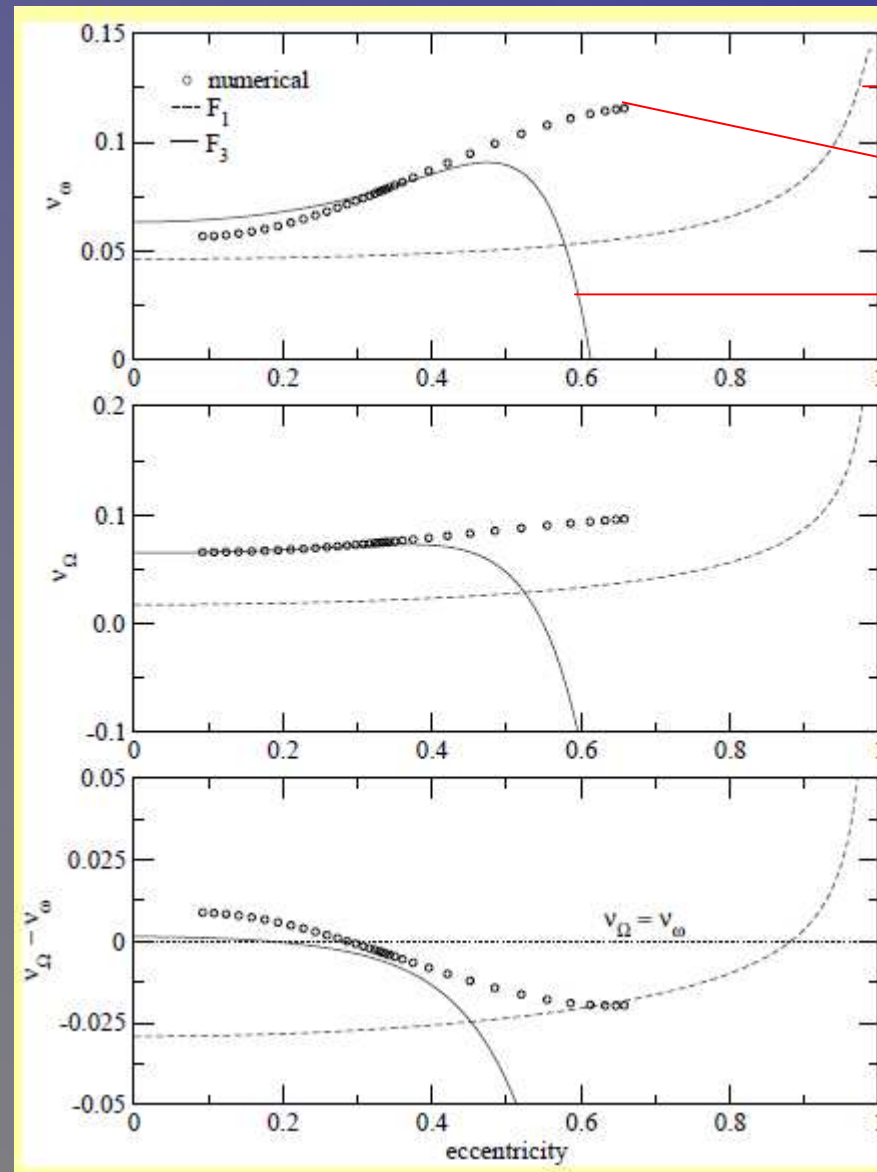


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Muchas gracias...

Frecuencias seculares



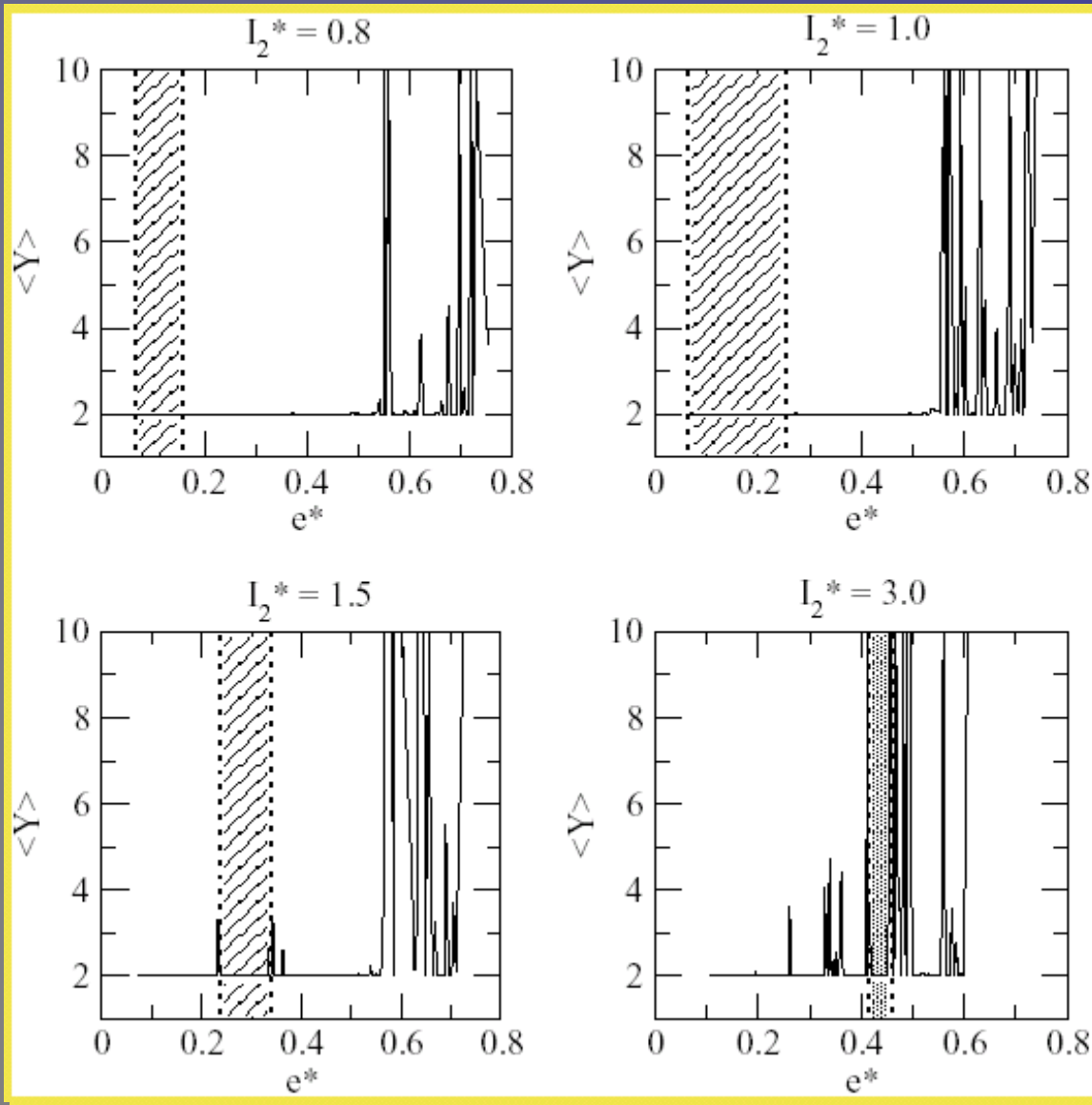
1^{er} ord.

Num.

3^{er} ord.



Mapas de la Resonancia... (Sinope)



$$a^* = 0.16UA$$

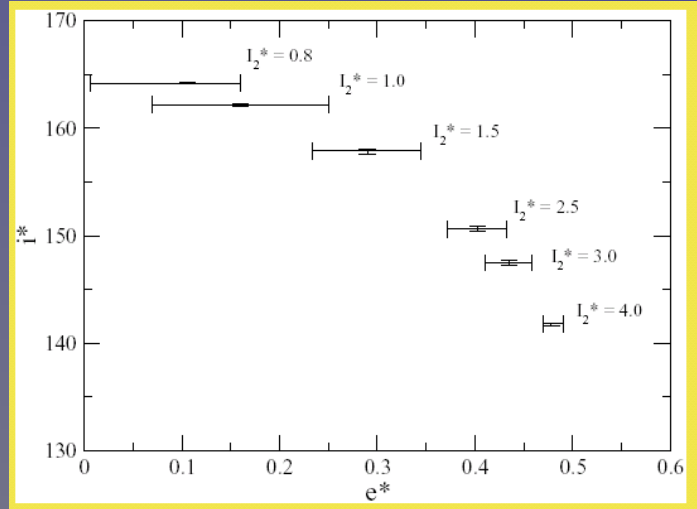
$$L^* = 2.115 \times 10^{-4}$$

$$I_1^* = -G^* = -L^* \sqrt{1 - e^{*2}}$$

$$I_2^* = G^* + H^* = I_1^* (1 + \cos i^*)$$

MR & INOS: $\langle Y \rangle$

Cincotta & Sussman (2009)







- **Introducción**
- **Modelos**
- **Mapas en un Planeta**
- **Familias de Soluciones Resonantes**
- **Relaciones entre Mapas**
- **Conclusiones**

Implementando el Modelo Numérico

elem. osc. a var. Res.

filtro

$$(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$



$$(I_1^*, \phi^*, I_2^*, L^*)$$

~~$$(M, M_{\alpha}, Q)$$~~

Constr. Mapa

$$(I_2^*, L^*)$$

?



$$(\bar{I}_2^*, \bar{L}^*)$$

$$L^* = L + \frac{\partial}{\partial M} \chi = f_1(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$I_2^* = I_2 + \frac{\partial}{\partial Q} \chi = f_2(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$L^* + \Delta L^* = L^* + \frac{\partial f_1}{\partial a} \Delta a + \frac{\partial f_1}{\partial i} \Delta i = \bar{L}^*$$

$$I_2^* + \Delta I_2^* = I_2^* + \frac{\partial f_2}{\partial a} \Delta a + \frac{\partial f_2}{\partial i} \Delta i = \bar{I}_2^*$$

$$\chi = \chi(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$

Motivación de su estudio

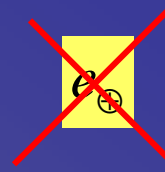
No está claro si la población resonante observada en otros planetas se debe a una migración satelital lenta

o simplemente son casos fortuitos

El criterio estadístico de proximidad a la resonancia que da la probabilidad de que estén en resonancia Beauge y Nesvorny permiten determinar la probabilidad de que sean.... y justo los casos de Pasiphae y Sinope no parecen ser casuales

modelos de alto orden no sirven

Principales efectos



- Cambio del valor del semieje mayor

$$a_{\alpha} \rightarrow a'_{\alpha}$$

$$\Gamma_a = \frac{a'_{\alpha}}{a_{\alpha}}$$



$$a^{*} = \Gamma_a a^{*}$$

- Cambio del valor de la masa del planeta
Aproximación de Hill (PC3CR)

$$\vec{r} \rightarrow \bar{\mu}^{\frac{1}{3}} \vec{r}$$

$$\Gamma_m = \left(\frac{m'_p}{m_p} \right)^{\frac{1}{3}}$$



$$m'_p = \Gamma_m^3 m_p$$

$$a^{*} = \Gamma_a \Gamma_m a^{*}$$