

Mapas de la Resonancia Secular ν_{\odot} para Satélites Irregulares Retrógrados

Correa Otto J., **Leiva A.M.**, Giuppone C.A., Beaugé C.

Observatorio Astronómico de Córdoba, U.N.C.

■ Introducción

Satélites Irregulares al 2/2010

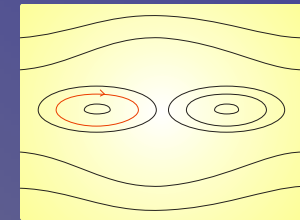
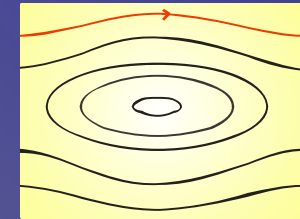
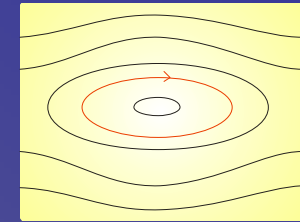
Júpiter 55

Saturno 38

Urano 9

Neptuno 6

$$\bar{\omega} - \bar{\omega}_\alpha$$



■ Whipple & Shelus (1993)

■ Saha & Tremaine (1993)

■ Nesvorný et al.(2003)

■ Cuk & Burns (2004)

■ Beaugé & Nesvorný (2007)

Pasiphae(J⁻)

Sinope(J⁻)

Pasiphae

Siarnaq(S⁺) – Sthepano(U⁻)

Narvi(S⁻)

libra (0°)

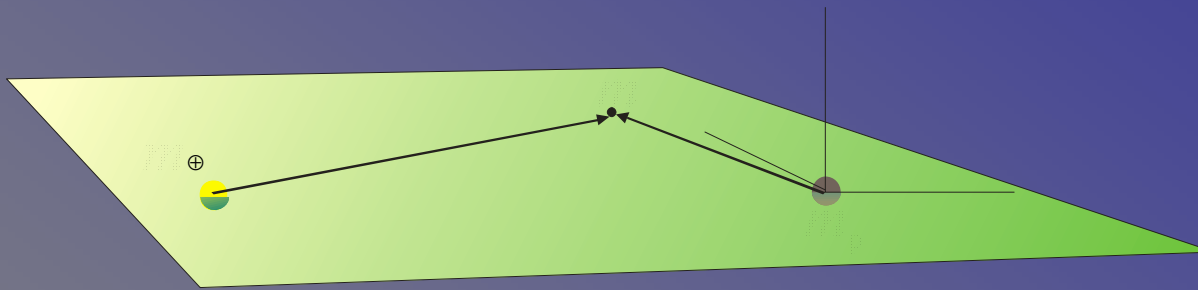
libra (0°)-circ.

circulación

libra (180°)

libración asimétrica

Modelos Analíticos para Resonancia...



Variables canónicas Delaunay

$$\begin{aligned} L &= \sqrt{\mu a} & ; & M \\ G &= L\sqrt{1-e^2} & ; & \omega \\ H &= G \cos i & ; & \Omega. \end{aligned}$$

$$F = F_0(L, \Lambda) + R(L, G, H, M, \omega, \Omega, M_{\alpha})$$

P2C

Perturbación

Yokoyama et al. (2003)

$$F_1(G^*, H^*, \omega^*, \Omega^*; \alpha^*)$$

$$\alpha^* = \frac{L^{*2}}{\mu a_{\alpha}}$$

variable de interés



$$\theta = \varpi - \varpi_{\alpha}$$

órbitas

directas

$$i < 90^{\circ}$$

$$\varpi = \Omega + \omega$$

órbitas

retrógradas

$$i > 90^{\circ}$$

$$\varpi = \Omega - \omega$$

variables “medias”

$$\begin{aligned} I_1 &= -G^* & ; & \quad \phi = \Omega^* - \omega^* \\ I_2 &= G^* + H^* & ; & \quad Q = \Omega^* \end{aligned}$$

$$\theta = \phi - \varpi_{\alpha}$$

variables “medias-medias”

T.C.

Nuevo Hamiltoniano

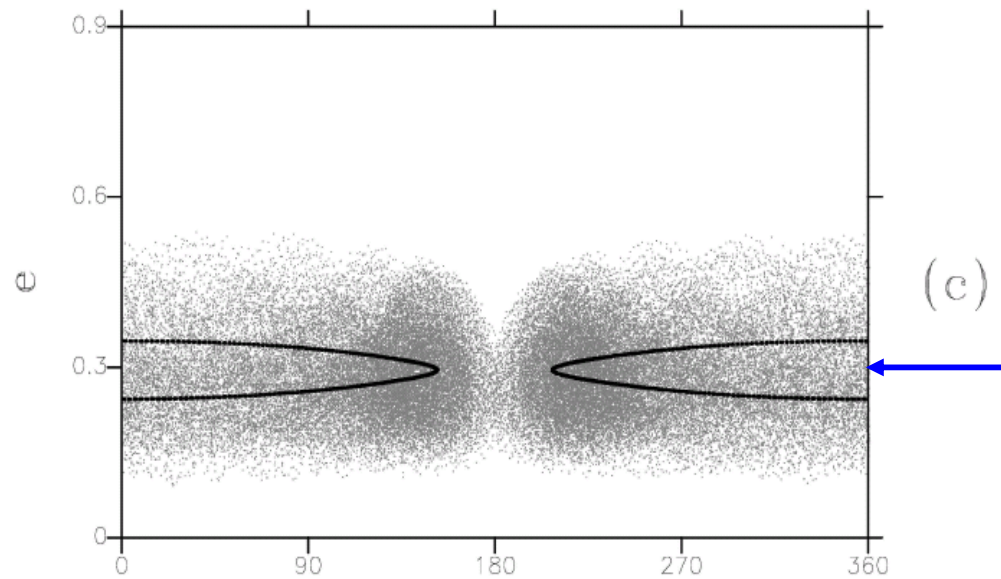
$$(I_1, I_2, \theta, Q) \rightarrow (I_1^*, I_2^*, \theta^*, Q^*)$$

$$F_1(I_1, I_2, \theta, Q) \rightarrow F_1^*(I_1^*, I_2^*, \theta^*)$$

$$F_1^* = A_0 + A_1 \cos(\theta^*) + A_2 \cos(2\theta^*)$$

$$A_k(I_1^*, I_2^*, L)$$

Dinámica secular de Sinope

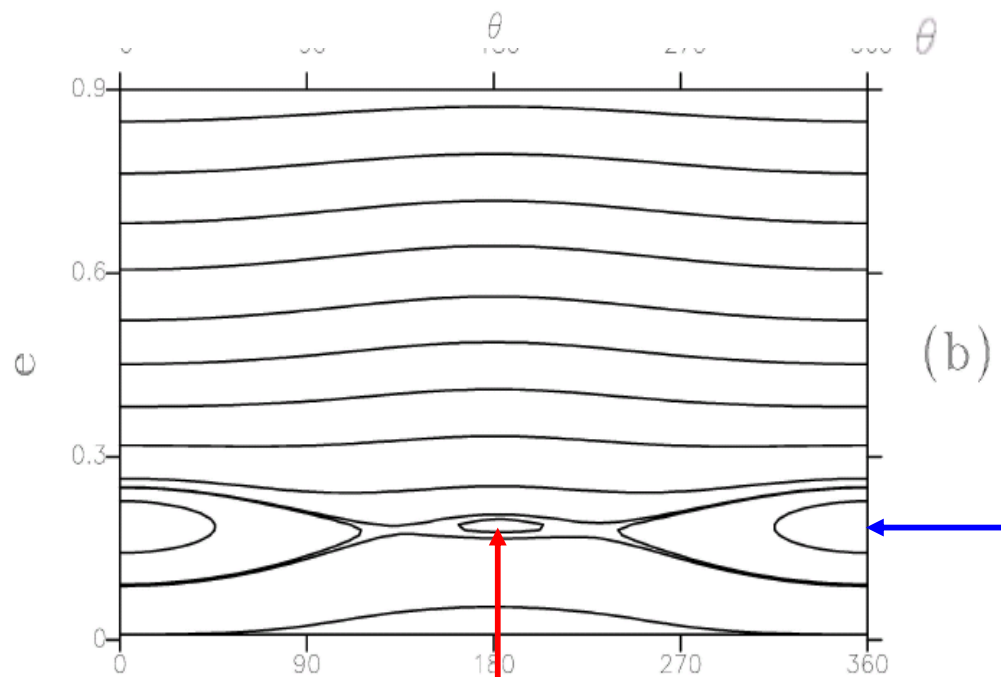


(c)

$$e^* = e^*(I_1^*, L^*, I_2^*)$$

$$L^* = 2.115 \times 10^{-4}$$

$$I_2^* = 1.575 \times 10^{-5}$$

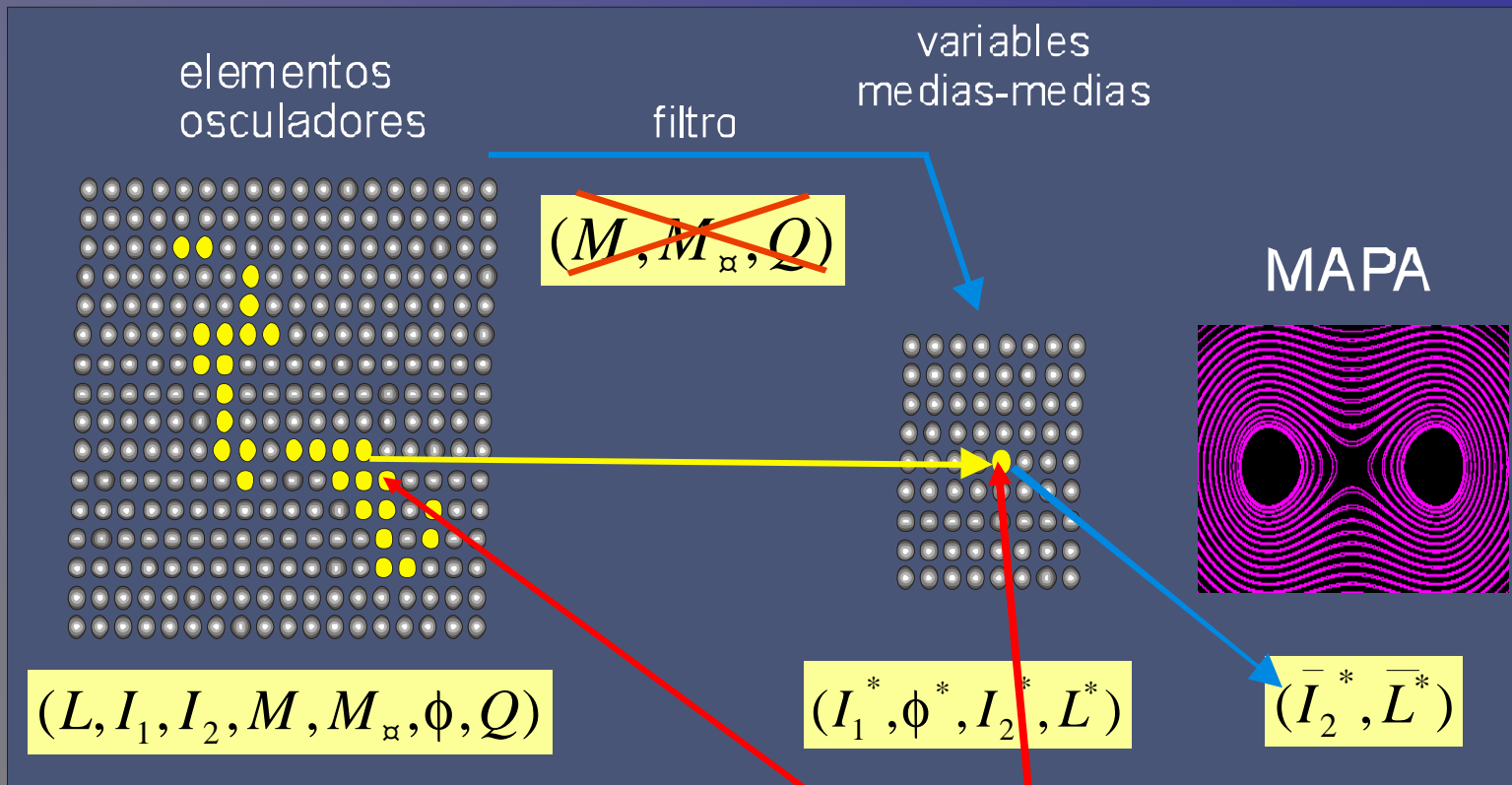


(b)

Beaugé & Nesvorný (2007)

$$F_3(G^*, H^*, \omega^*, \Omega^*; \alpha^*)$$

Implementando el Modelo Numérico



$$L^* = L + \frac{\partial}{\partial M} \chi = f_1(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

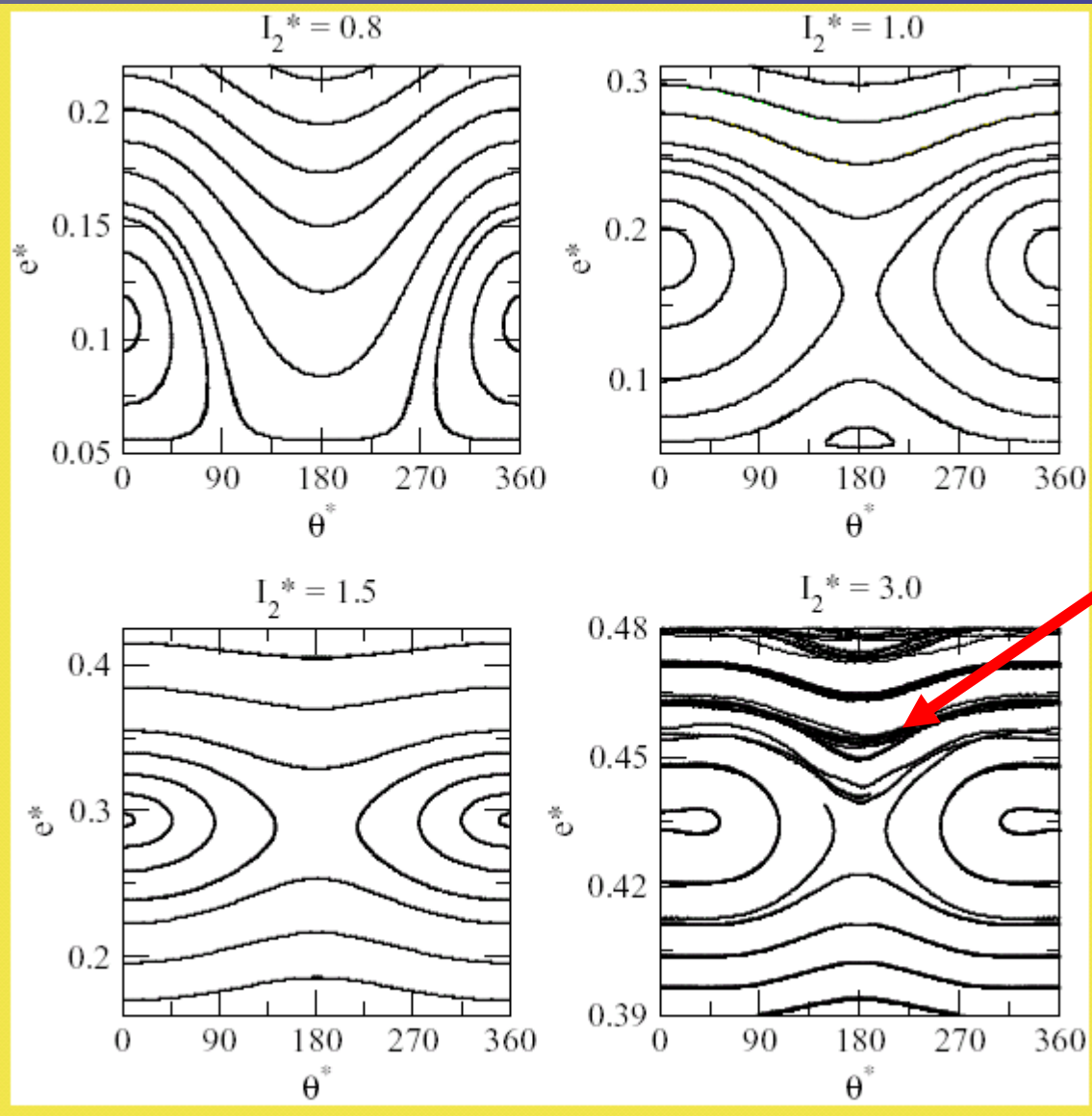
$$I_2^* = I_2 + \frac{\partial}{\partial Q} \chi = f_2(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$\chi = \chi(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$

$$L^* + \Delta L^* = L^* + \frac{\partial f_1}{\partial a} \Delta a + \frac{\partial f_1}{\partial i} \Delta i = \bar{L}^*$$

$$I_2^* + \Delta I_2^* = I_2^* + \frac{\partial f_2}{\partial a} \Delta a + \frac{\partial f_2}{\partial i} \Delta i = \bar{I}_2^*$$

Mapas de la Resonancia... (Sinope)



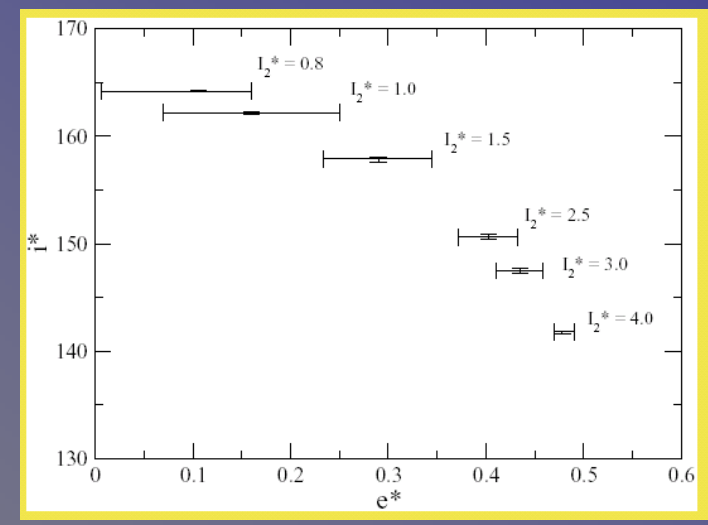
$$a^* = 0.16UA$$

$$L^* = 2.115 \times 10^{-4}$$

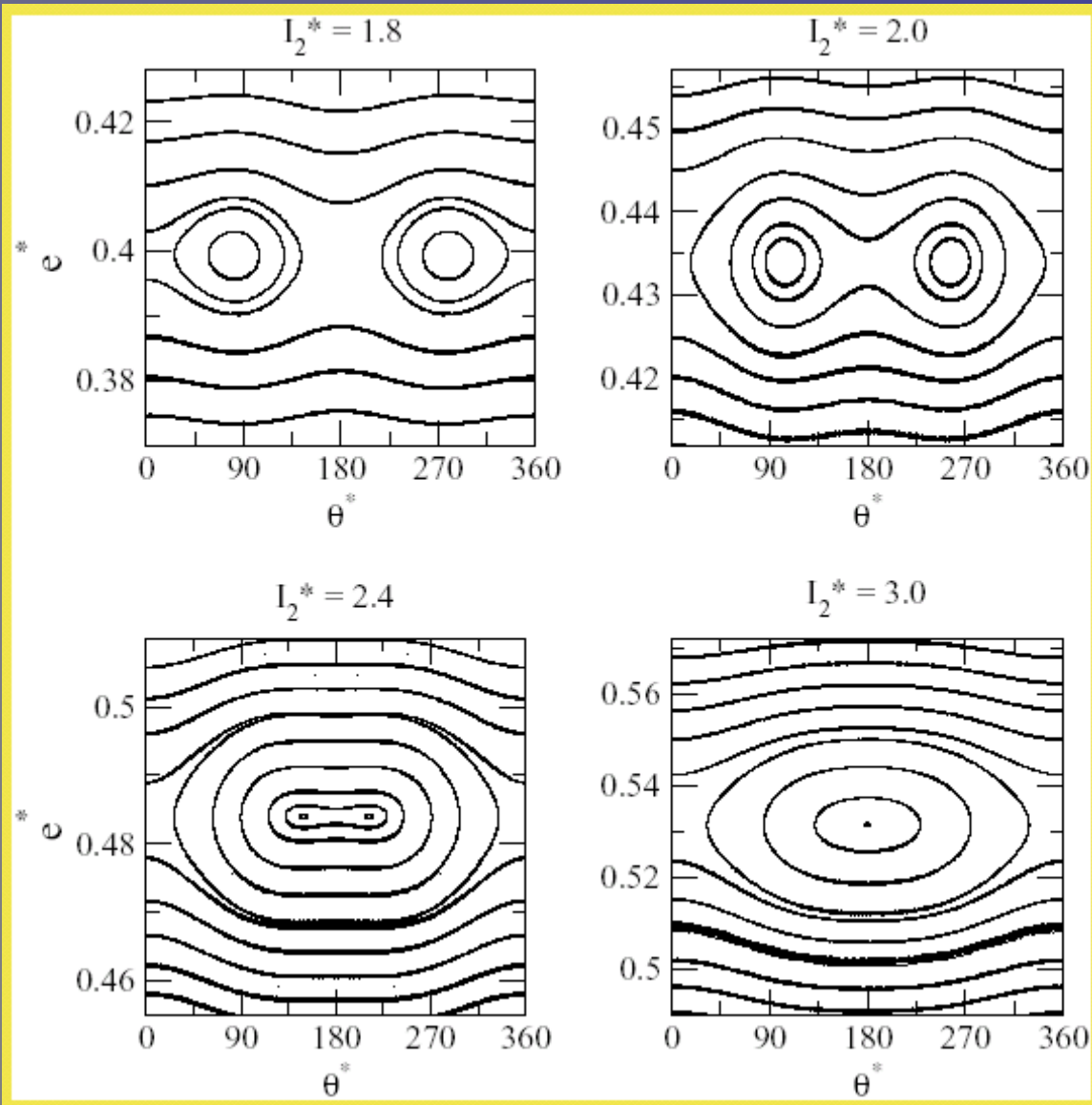
$$I_1^* = -G^* = -L^* \sqrt{1 - e^{*2}}$$

$$I_2^* = G^* + H^* = I_1^* (1 + \cos i^*)$$

RMM 6:1
Hinse et al. (2009)



Mapas de la Resonancia en Saturno...

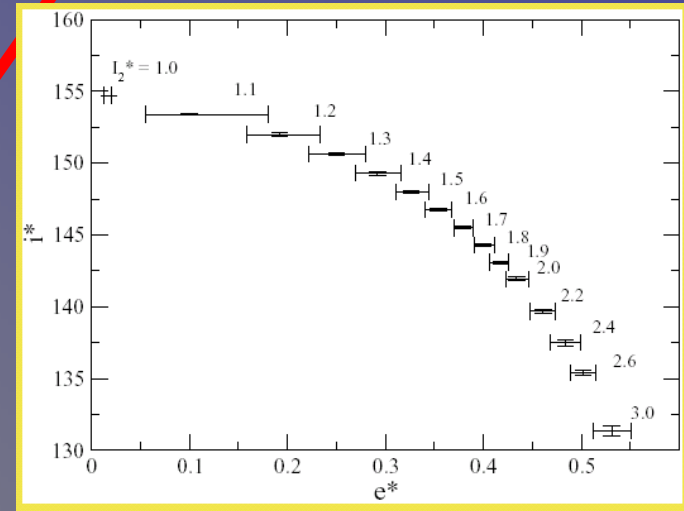


Narvi

$$a^* = 0.13UA$$

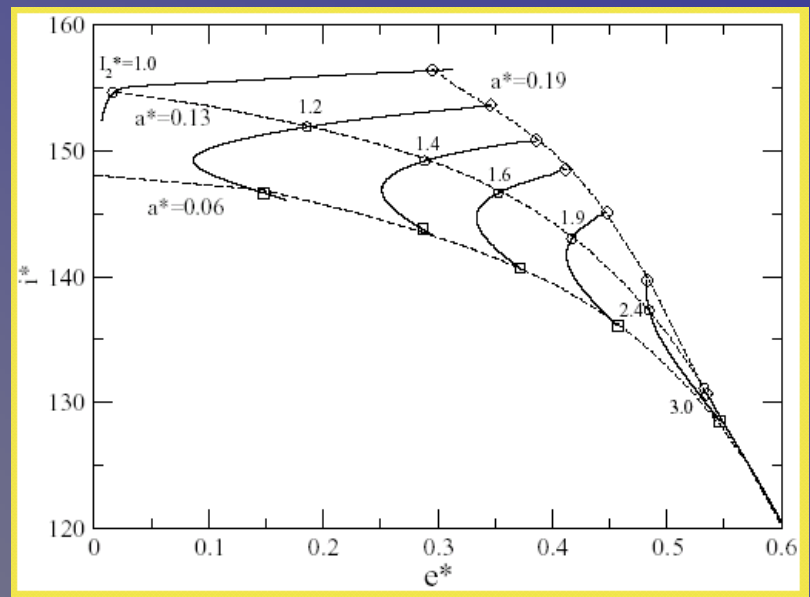
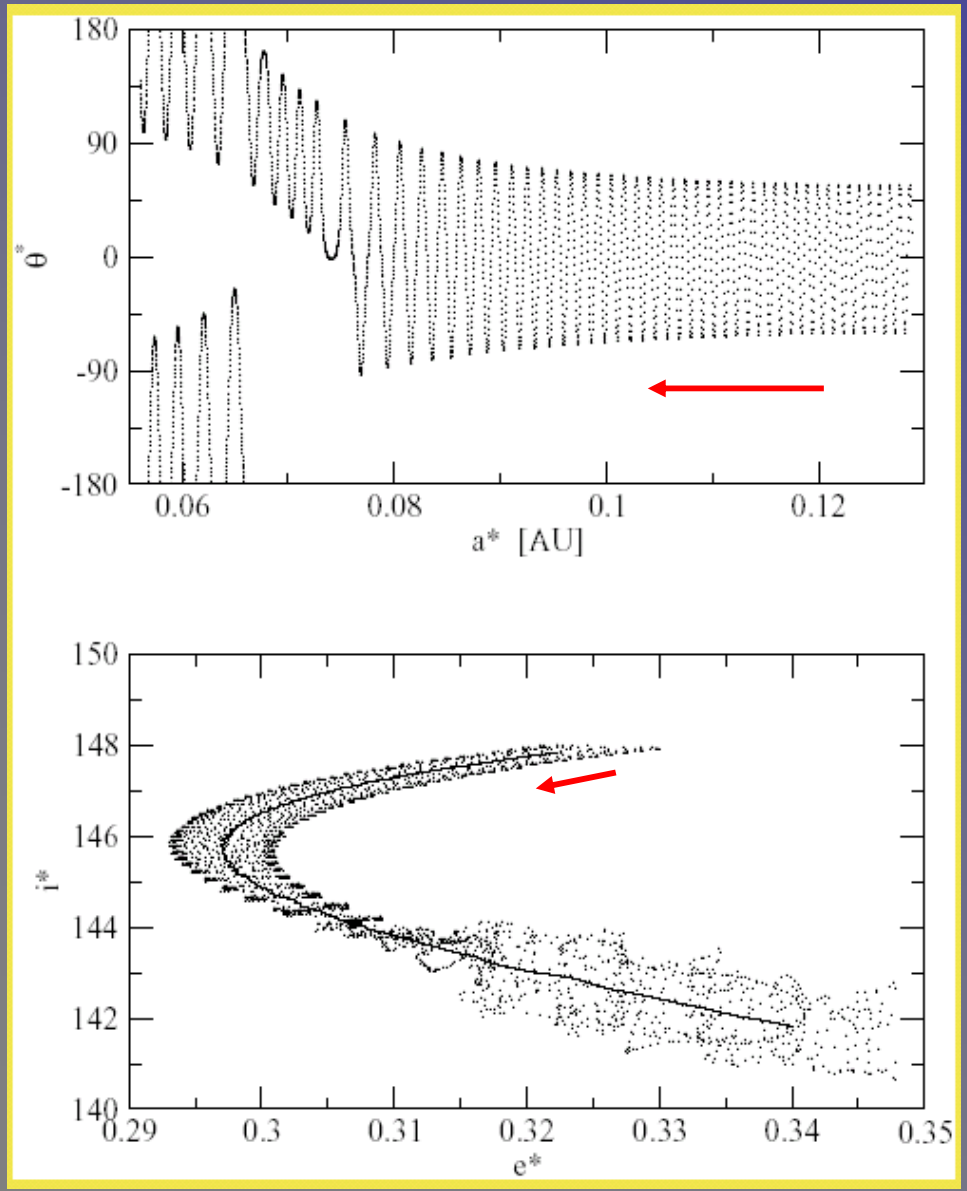
$$L^* = 1.040 \times 10^{-4}$$

libraciones
asimétricas

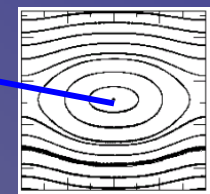
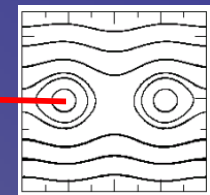
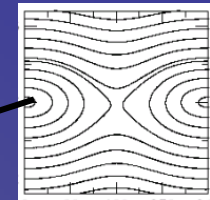
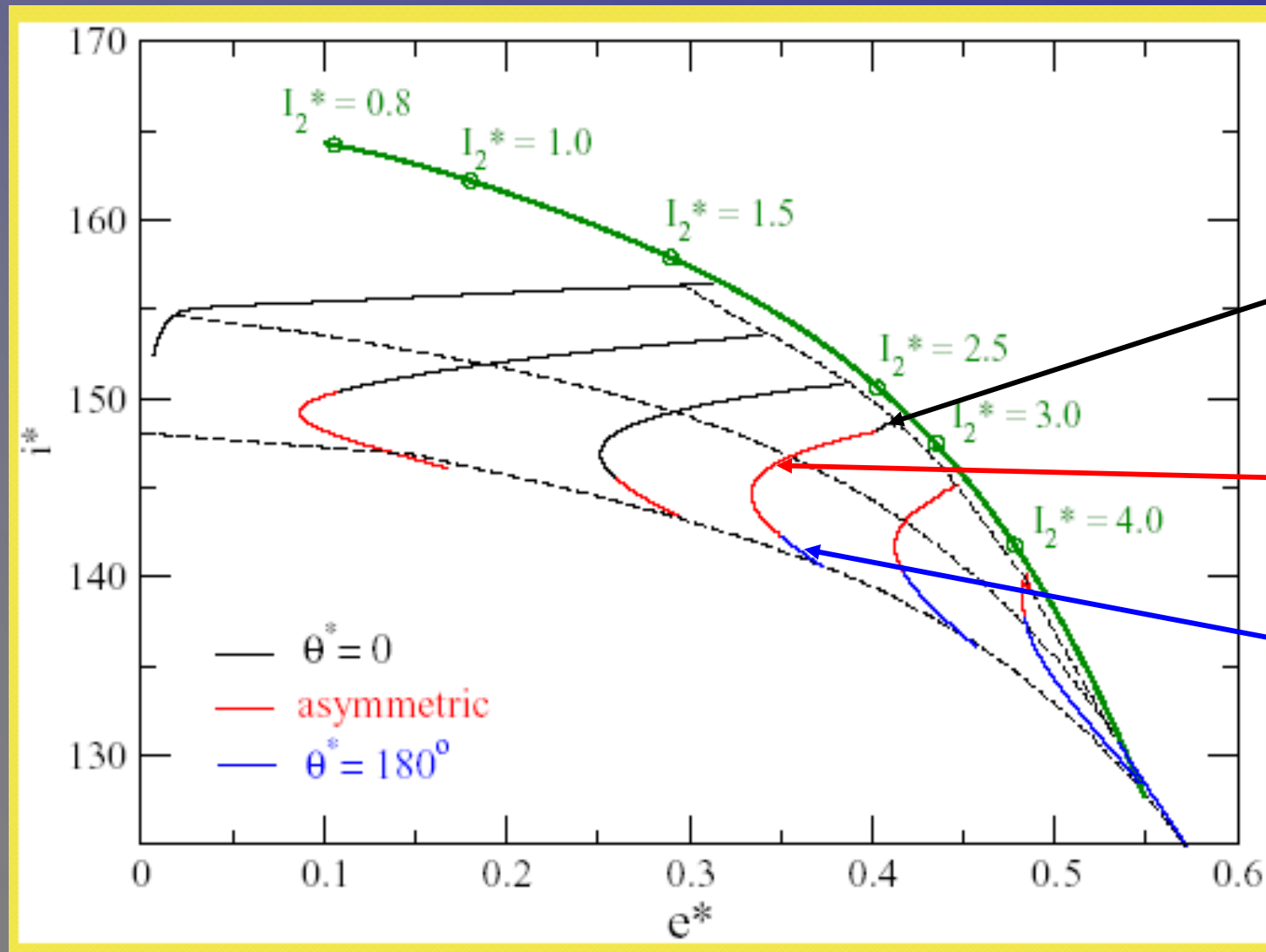


Utilizando migración adiabática

a^* $\tau_a = 10^7$ años



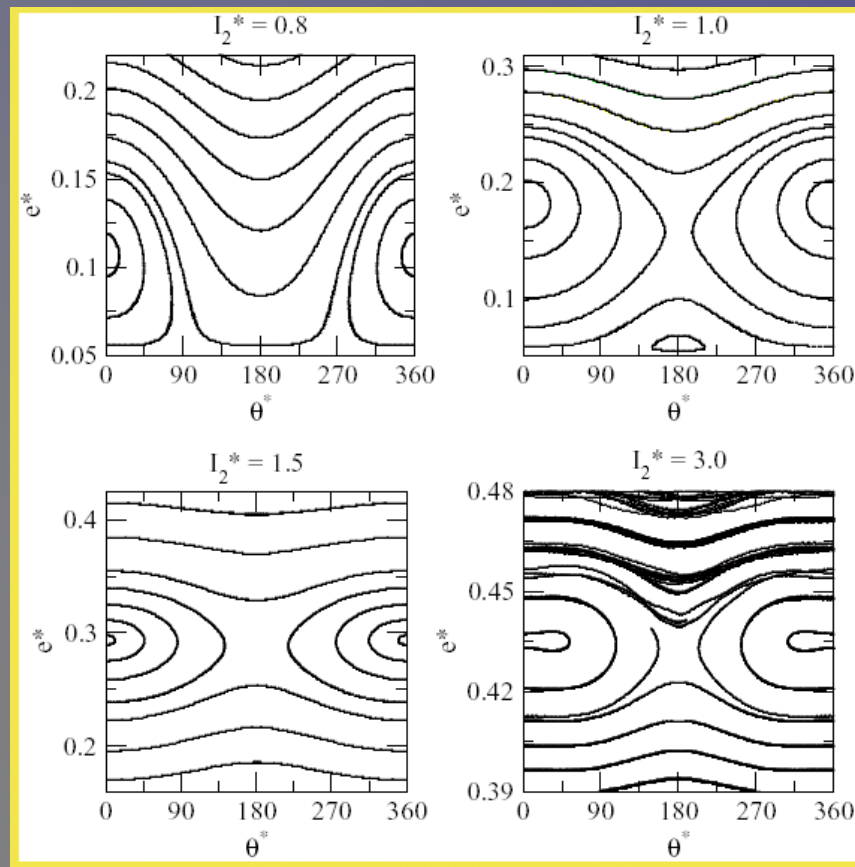
Familias de soluciones resonantes de amplitud-cero



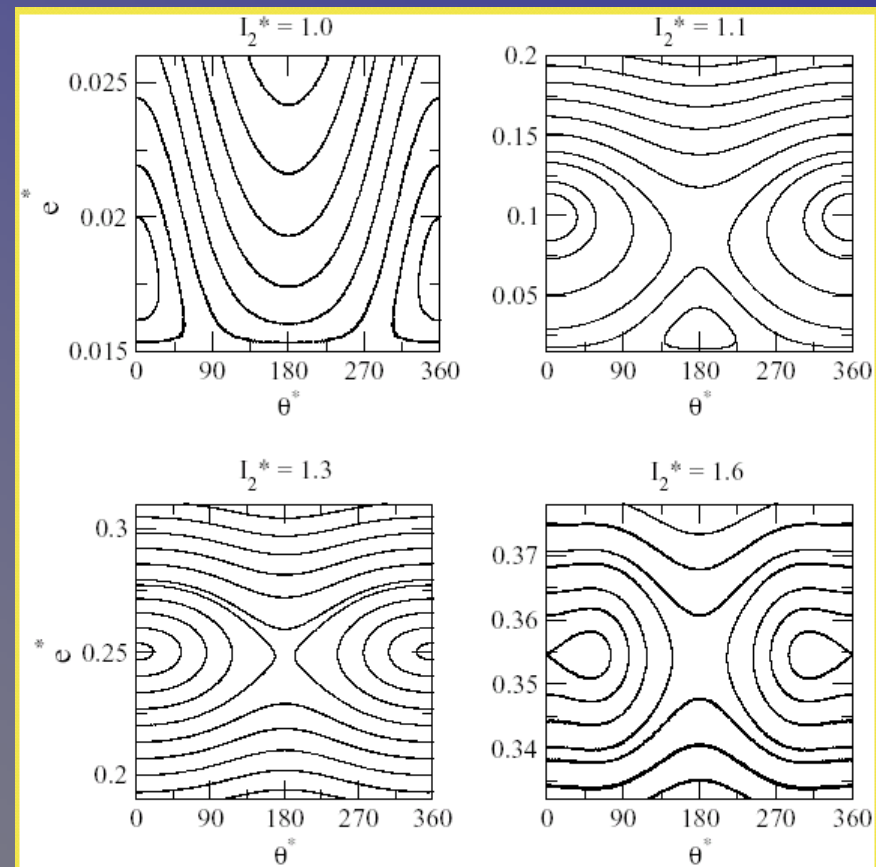
Factor de escala para otros sistemas de satélites

Teoría del Globo...

Júpiter



Saturno

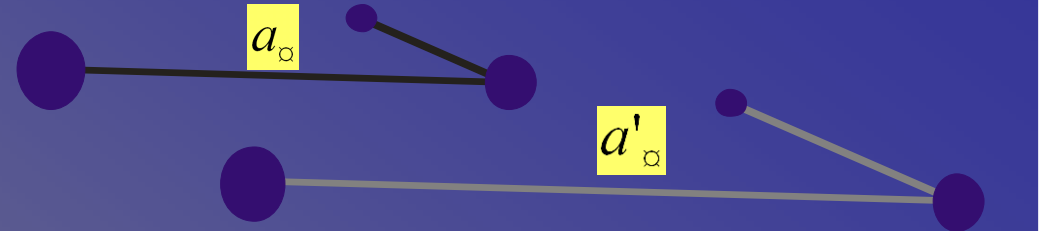


Principales efectos

■ Escala de distancias

$$\frac{a^*}{a_{\alpha}} \rightarrow \frac{a'^*}{a'_{\alpha}}$$

Las ecuaciones de movimiento quedan invariantes...



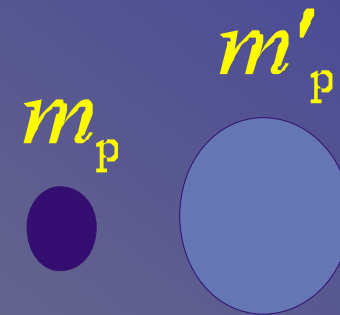
$$\Gamma_a = \frac{a'_{\alpha}}{a_{\alpha}}$$



$$a'^* = \Gamma_a a^*$$

■ Factor de masa Aproximación de Hill

$$\vec{r} \rightarrow \bar{\mu}^{\frac{1}{3}} \vec{r}$$



$$\Gamma_m = \left(\frac{m'_p}{m_p} \right)^{\frac{1}{3}}$$

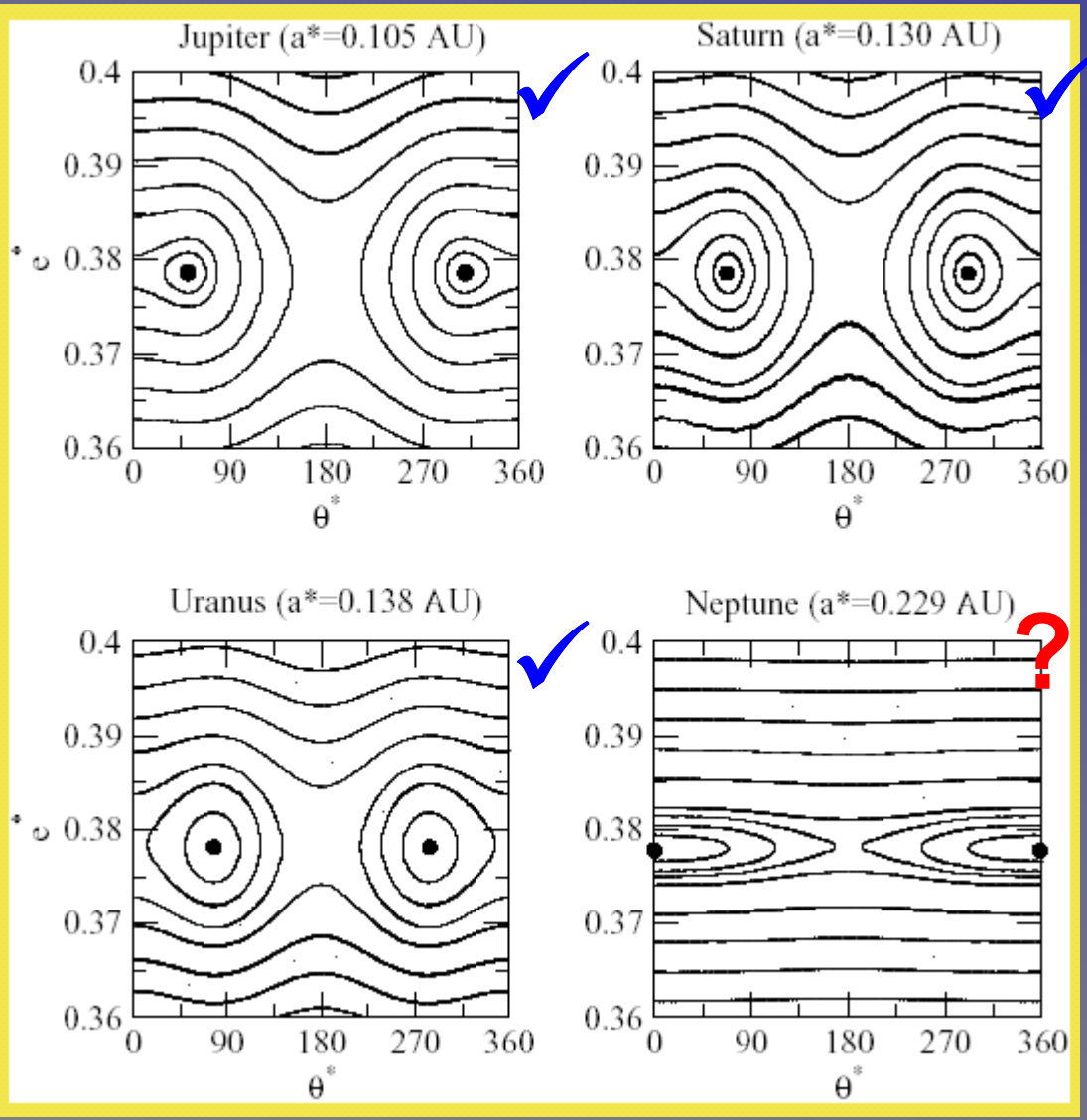


$$m'_p = \Gamma_m^3 m_p$$

FACTOR GLOBAL

$$a'^* = \Gamma_a \Gamma_m a^*$$

Resultados....



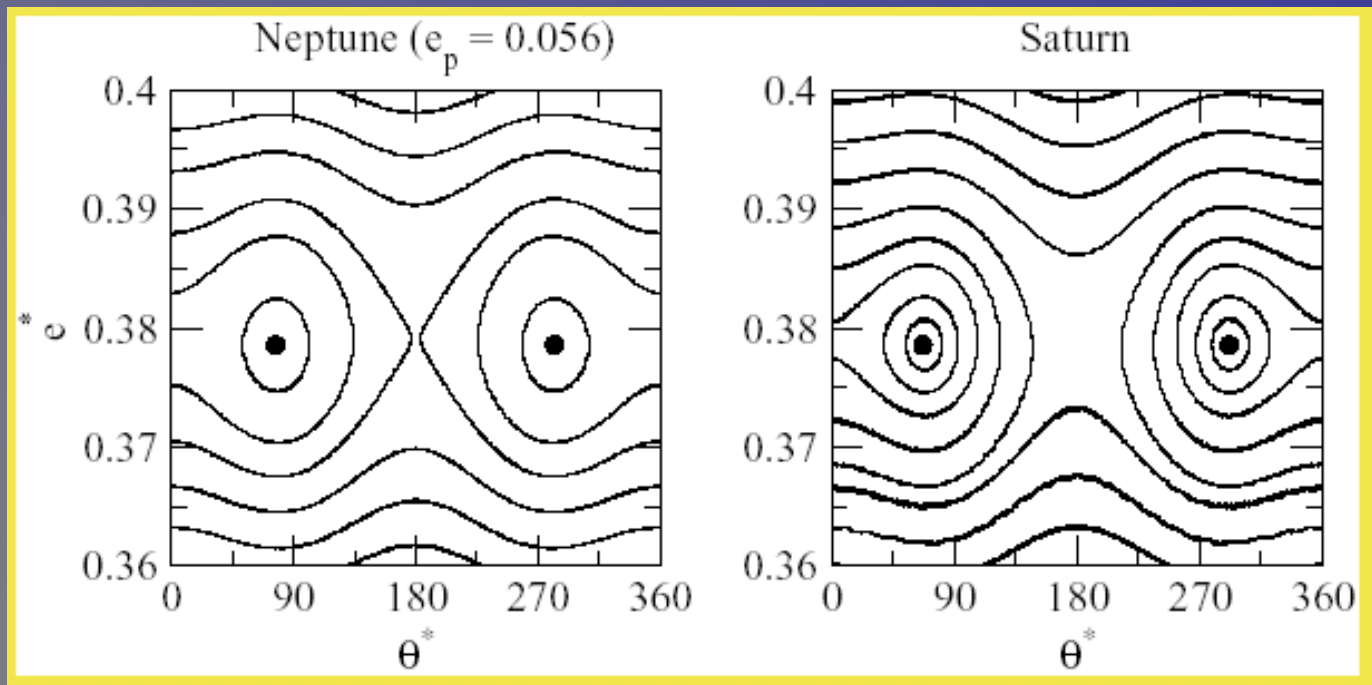
$$e_{Sat} > e_{Jup} > e_{Ura} > e_{Nep}$$

$$F_1^* = C_0 + C_1 e_{\alpha} \cos \theta^* + C_2 e_{\alpha}^2 \cos 2\theta^*$$



$$\sin \theta^* (C_1 + 4C_2 e_{\alpha} \cos \theta^*) = 0$$

$$4|C_2| e_{\alpha} > |C_1|$$



Conclusiones

- Dominio resonante con diversos modos de libración
- Modos restringidos a ciertos intervalos de i^* media-media

$$\theta = 0^0 \quad (i \geq 150^0)$$

$$\theta = 180^0 \quad (i \leq 140^0)$$

libraciones asimétricas en valores intermedios

- Buen acuerdo con la realidad (Pasiphae-Sinope-Narvi)
- Presentación de familias de soluciones resonantes (migración)
- Factor de escala de Mapas.

- Problema de Tres Cuerpos Restringido
- satélites retrógrados

Muchas gracias...

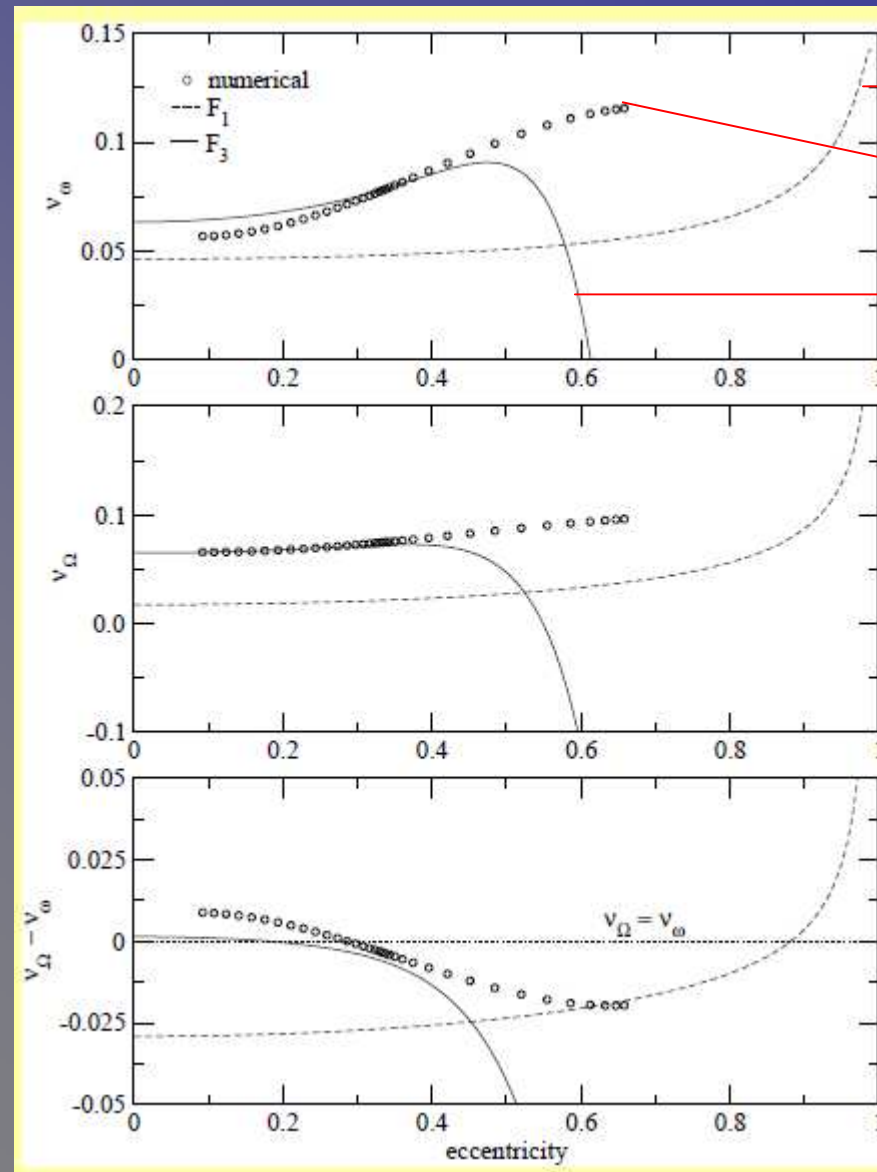


Referencias

- Beaugé, C. (1994). *CeMDA*, **60**, 225.
- Beaugé, C., Michtchenko, T.A. & Ferraz-Mello, S. (2006). *MNRAS*, **365**, 1160.
- Beaugé, C., Nesvorný, D. & Dones, L. (2006). *AJ*, **131**, 2299.
- Beaugé, C. & Nesvorný, D. (2007). *AJ*, **133**, 2537.
- Carpino, M., Milani, A. & Nobili, A.M. (1987). *A & A*, **181**, 182.
- Carruba, V., Nesvorný, D., Burns, J. A., Čuk, M. & Tsiganis, K. (2004). *AJ*, **128**, 1899.
- Cincotta, P.M., & Simó, C. (2000). *A & A*, **147**, 205.
- Čuk, M. & Burns, J.A. (2004). *AJ*, **128**, 2518.
- Hill, G.W. (1878). *Am. J. Math.*, **1**, 5.
- Hill, G.W. (1886). *Am. J. Math.*, **8**, 1.
- Hinse, T.C., Christou, A.A. & Alvarellos, J.L.A. (2009). *MNRAS*, submitted.
- Kaula, W.M. (1962). *AJ*, **67**, 300.
- Kozai, Y. (1962). *AJ*, **67**, 591.
- Lidov, R. (1961). Analiz Evolucii Orbit Iskustvennich Sputnikov. Problemi Dvigenia Iskustvennich Nebesnich Tel. Izd. Akad. Nauk SSSR, Moscow (1963), pp. 119.
- Murray, C.D. & Dermott, S.F. (1999). "Solar System Dynamics", Cambridge University Press, Chapter 6.
- Nesvorný, D., Alvarellos, J.L.A., Dones, L. & Levison, H.F. (2003). *AJ*, **126**, 398.
- Saha, P. & Tremaine, S. (1993). *Icarus*, **106**, 549.
- Szebehely, V. (1967). "Theory of Orbits", Academic Press, NY.
- Whipple, A. & Shelus, P.J. *Icarus*, **101**, 265.
- Wintner, A. (1941). "The Analytical Foundations of Celestial Mechanics". Princeton, N.J., Princeton University Press.
- Yokoyama, T., Santos, M.T., Cardin, G. & Winter, O.C. (2003). *A&A*, **401**, 763.

Muchas gracias...

Frecuencias seculares



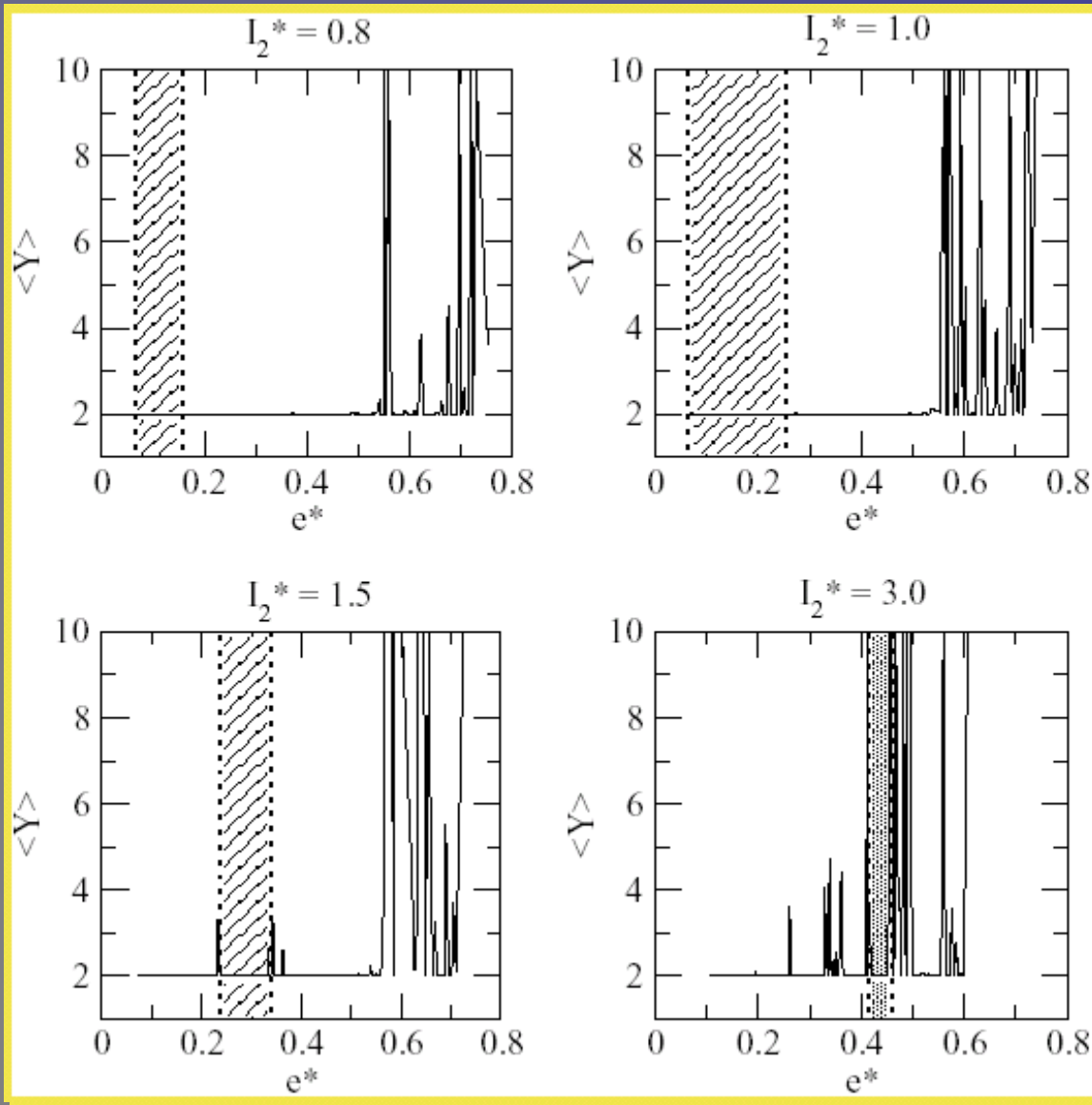
1^{er} ord.

Num.

3^{er} ord.



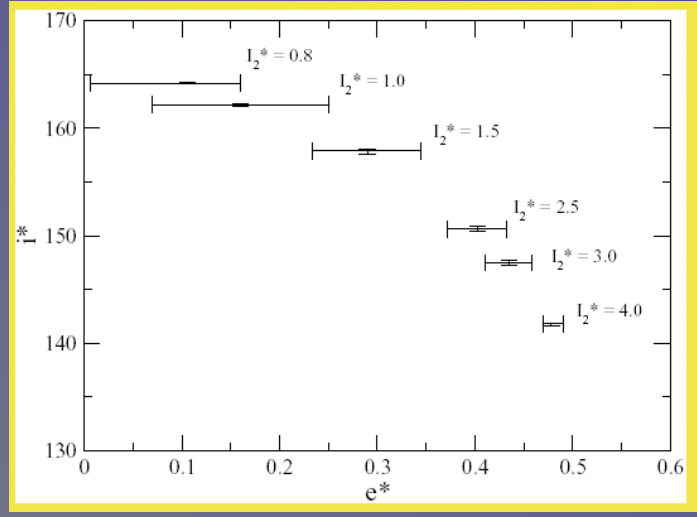
Mapas de la Resonancia... (Sinope)



$a^* = 0.16UA$
 $L^* = 2.115 \times 10^{-4}$
 $I_1^* = -G^* = -L^* \sqrt{1 - e^{*2}}$
 $I_2^* = G^* + H^* = I_1^* (1 + \cos i^*)$

MR & INOS: $\langle Y \rangle$

Cincotta et al (2009) (2000)







- **Introducción**
- **Modelos**
- **Mapas en un Planeta**
- **Familias de Soluciones Resonantes**
- **Relaciones entre Mapas**
- **Conclusiones**

Implementando el Modelo Numérico

elem. osc. a var. Res.

filtro

$$(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$



$$(I_1^*, \phi^*, I_2^*, L^*)$$

~~$$(M, M_{\alpha}, Q)$$~~

Constr. Mapa

$$(I_2^*, L^*)$$

?



$$(\bar{I}_2^*, \bar{L}^*)$$

$$L^* = L + \frac{\partial}{\partial M} \chi = f_1(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$I_2^* = I_2 + \frac{\partial}{\partial Q} \chi = f_2(a, e, i, M, M_{\alpha}, \omega, \Omega)$$

$$L^* + \Delta L^* = L^* + \frac{\partial f_1}{\partial a} \Delta a + \frac{\partial f_1}{\partial i} \Delta i = \bar{L}^*$$

$$I_2^* + \Delta I_2^* = I_2^* + \frac{\partial f_2}{\partial a} \Delta a + \frac{\partial f_2}{\partial i} \Delta i = \bar{I}_2^*$$

$$\chi = \chi(L, I_1, I_2, M, M_{\alpha}, \phi, Q)$$

Motivación de su estudio

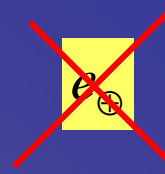
No está claro si la población resonante observada en otros planetas se debe a una migración satelital lenta

o simplemente son casos fortuitos

El criterio estadístico de proximidad a la resonancia que da la probabilidad de que estén en resonancia Beauge y Nesvorny permiten determinar la probabilidad de que sean.... y justo los casos de Pasiphae y Sinope no parecen ser casuales

modelos de alto orden no sirven

Principales efectos



- Cambio del valor del semieje mayor

$$a_{\alpha} \rightarrow a'_{\alpha} \quad \Gamma_a = \frac{a'_{\alpha}}{a_{\alpha}} \quad \longrightarrow \quad a'^{*} = \Gamma_a a^*$$

- Cambio del valor de la masa del planeta
Aproximación de Hill (PC3CR)

$$\vec{r} \rightarrow \bar{\mu}^{\frac{1}{3}} \vec{r} \quad \Gamma_m = \left(\frac{m'_p}{m_p} \right)^{\frac{1}{3}} \quad \longrightarrow \quad m'_p = \Gamma_m^3 m_p$$

$$a'^{*} = \Gamma_a \Gamma_m a^*$$